Does the Market Understand Time Variation in the Equity Premium?

MIHIR GANDHI
Chicago Booth

NIELS J. GORMSEN
Chicago Booth

EBEN LAZARUS
MIT Sloan

OCTOBER 2022

Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?

Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?

$$p_t - d_t = \kappa - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

$$\mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}$$

much more important for pricing!

Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?

$$p_t - d_t = \kappa - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}}_{\mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

Our focus:

- What is the **expected future equity premium**?
- How does it compare to the *actual* future equity premium $\mathbb{E}_{t+j} r_{t+j+1}$?
- Are there systematic errors in *expected* return predictions?

What We Do

1. Option-based measure of log equity premium:

$$\textbf{Spot rate:} \qquad \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}] - r^f_{t,t+n}$$

2. Calculate expected future equity premium:

$$\textbf{Forward rate:} \qquad f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}]$$

3. Compare forward rate to realized future spot rate:

$$\textbf{Forecast error:} \qquad \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)}$$

Why This Framework?

$$\begin{aligned}
\textbf{Spot rate:} \quad & \mu_t^{(n)} &&= \mathbb{E}_t[r_{t,t+n}] - r^f_{t,t+n} \\
\textbf{Forward rate:} \quad & f_t^{(n)} &&= \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \\
\textbf{Forecast error:} \quad & \varepsilon_{t+n} &&= \mu_{t+n}^{(1)} - f_t^{(n)}
\end{aligned}$$

- Term structure of **log** equity premia $\Longrightarrow$ straightforward calculation of forward rates
- Behavior of term structure:
  - Key for prices
  - Great lab for testing whether market-based expectations are intertemporally consistent, *without* needing to take a stand on whether expected returns are themselves rational
- Forecast errors require much weaker conditions than needed to estimate $\mu_t^{(n)}$ and $f_t^{(n,m)}$ separately
  - In general, spot and forward rates are contaminated by SDF-related covariances. . .
  - . . .but when differencing, the covariance terms largely cancel

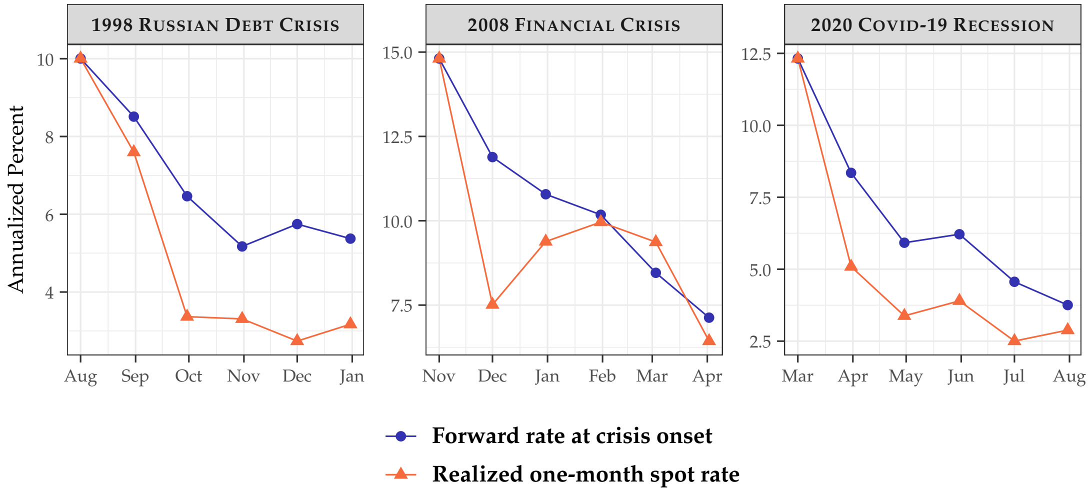
What We Find

$$\begin{aligned} \textbf{Spot rate:} \quad \mu_t^{(n)} &= \mathbb{E}_t[r_{t,t+n}] - r^f_{t,t+n} \\ \textbf{Forward rate:} \quad f_t^{(n)} &= \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \\ \textbf{Forecast error:} \quad \varepsilon_{t+n} &= \mu_{t+n}^{(1)} - f_t^{(n)} \end{aligned}$$

In global options sample:

1. Forward rates are strong predictors of realized spot rates
   - Forward rate $\nearrow$ by **1%** $\Longrightarrow$ future spot rate $\nearrow$ by **0.7%**
   - Forward rates explain **20%** of the variation in future spot rates at 6-month horizon
2. Forecast errors are **close to 0** on average. . .
3. . . .but still exhibit predictable mean reversion and excess volatility
   - Can therefore reject fully rational expectations for range of possible SDF specifications
   - Alternatively, need highly volatile and countercyclical price of discount-rate risk

Illustration: U.S. Forward and Realized Spot Rates in Three Crises



Roadmap

The Log Utility Case: Setup

Setting:

- Representative agent ("the market")
  - Equivalently, any unconstrained investor who chooses to fully invest in the market
- For now: **Log utility** over wealth

$$\log(W_{t+n}) = \log(W_t R_{t,t+n})$$

$$P_t = \mathbb{E}_t[M_{t,t+n} \text{Payoff}_{t+n}]$$

$$M_{t,t+n} = 1/R_{t,t+n}$$

  - Useful for exposition, but will turn out not to be central

The Log Utility Case: Setup

Setting:

- Representative agent ("the market")
  - Equivalently, any unconstrained investor who chooses to fully invest in the market
- For now: **Log utility** over wealth $\Longleftrightarrow M_{t,t+n} = 1/R_{t,t+n}$
- **Building block**: **LVIX** $\mathcal{L}_t^{(n)}$ [Gao and Martin (2021)]:

$$\underbrace{\mathbb{E}_t[r_{t,t+n}] - r^f_{t,t+n}}_{\mu_t^{(n)}} = \underbrace{(R^f_{t,t+n})^{-1}\mathbb{E}^*_t[R_{t,t+n}r_{t,t+n}] - r^f_{t,t+n}}_{\mathcal{L}_t^{(n)}} - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})$$

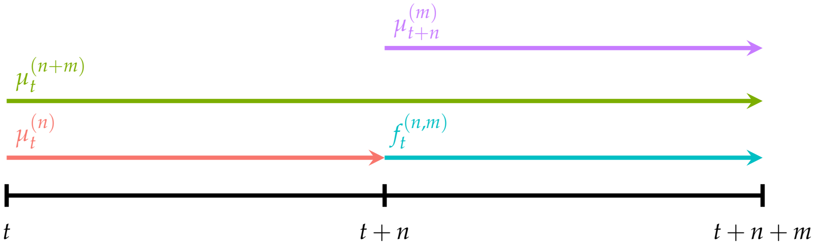
  - $\mathcal{L}_t^{(n)}$: Observable from options (details later)
  - Second term: $MR = 1 \Longrightarrow \text{Cov} = 0$ under log utility

The Log Utility Case: Result

General notation:

Spot rate:	$\mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}] - r^f_{t,t+n}$
Forward rate:	$f_t^{(n,m)} = \mu_t^{(n+m)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(m)}]$
Forecast error:	$\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}$

Timeline:



The Log Utility Case: Result

General notation:

$$\textbf{Spot rate:} \quad \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}] - r^f_{t,t+n}$$

$$\textbf{Forward rate:} \quad f_t^{(n,m)} = \mu_t^{(n+m)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(m)}]$$

$$\textbf{Forecast error:} \quad \varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}$$

Result 1 (*Log Utility Identification*)

Given $M_{t,t+n} = 1/R_{t,t+n}$:

$$\mu_t^{(n)} = \mathcal{L}_t^{(n)}$$

$$f_t^{(n,m)} = \mathcal{L}_t^{(n+m)} - \mathcal{L}_t^{(n)}$$

$$\varepsilon_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_t^{(n+m)} + \mathcal{L}_t^{(n)}$$

- Can then examine term structure, test $\mathbb{E}[\varepsilon_{t+n}^{(m)}] = 0$, $\mathbb{E}[Z_t \varepsilon_{t+n}^{(m)}] = 0, \ldots$

The General Case: Identification Challenge

Beyond log utility?

- In general, spot and forward rates are contaminated by unobservable covariances:

$$\mu_t^{(n)} = \mathscr{L}_t^{(n)} - \overbrace{\mathrm{Cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})}^{\mathcal{C}_t^{(n)}}$$

$$f_t^{(n,m)} = \mathscr{L}_t^{(n+m)} - \mathscr{L}_t^{(n)} + \mathcal{C}_t^{(n)} - \mathcal{C}_t^{(n+m)}$$

- For $\mu_t^{(n)}$, can argue $\mathcal{C}_t^{(n)} \leqslant 0$ [Gao & Martin (2021)]...but for $f_t^{(n,m)}$, $\mathcal{C}_t^{(n+m)} \overset{??}{\gtrless} \mathcal{C}_t^{(n)}$

- Key insight: Covariance terms largely cancel when considering **forecast errors** $\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}$

- Under log-normality, for $n = m = 1$:

$$\mathbb{E}_t[\mathcal{C}_{t+1}^{(1)}] + \mathcal{C}_t^{(1)} - \mathcal{C}_t^{(2)} = \mathrm{Cov}_t\big(M_{t,t+1}R_{t,t+1}, \underbrace{\mathbb{E}_{t+1}[r_{t+1,t+2}]}_{\textit{much less volatile than } r_{t+1,t+2}}\big)$$

The Log-Normal Case: Result

Define forecast-error proxy from log utility case:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathscr{L}_{t+n}^{(m)} - \mathscr{L}_{t}^{(n+m)} + \mathscr{L}_{t}^{(n)}$$

To continue building toward general case...

Result 2 (*Log-Normal Identification*)

For a general SDF $M_{t,t+n}$, assuming $M_{t,t+n}, R_{t,t+n}$ are jointly log-normal:

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])$$

- Covariance term now relates to pricing of *discount-rate risk*, rather than *realized-return* risk
- Basic idea of proof: $MR_{t,t+n}$ is orthogonal to unexpected component of $r_{t+n,t+n+m}$, so left with expectation term $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$
- Remaining covariance is likely quite small, but can be disciplined empirically or theoretically

The General Case: Result

Define forecast-error proxy and expected-return proxy:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t}^{(n+m)} + \mathcal{L}_{t}^{(n)}$$

$$\widehat{\mu}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} + r^{f}_{t+n,t+n+m}$$

Result 3 (*Generalized Identification*)

For any SDF $M_{t,t+n}$,

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \text{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \widehat{\mu}_{t+n}^{(m)}\right)$$

- Intuition from log-normal case carries over, with $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ replaced by $\widehat{\mu}_{t+n}^{(m)}$
- LVIX-based $\widehat{\mu}_{t+n}^{(m)}$ is closely related to $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ ...but $\widehat{\mu}_{t+n}^{(m)}$ is directly observable
- Main specification: $\widehat{\mu}_{t+n}^{(m)}$ is $\frac{1}{10}$ as volatile as realized return $r_{t+n,t+n+m}$

  $\Longrightarrow$ unobserved covariance likely much smaller for forecast errors than for spot rates

Roadmap

Data and Measurement

Data:

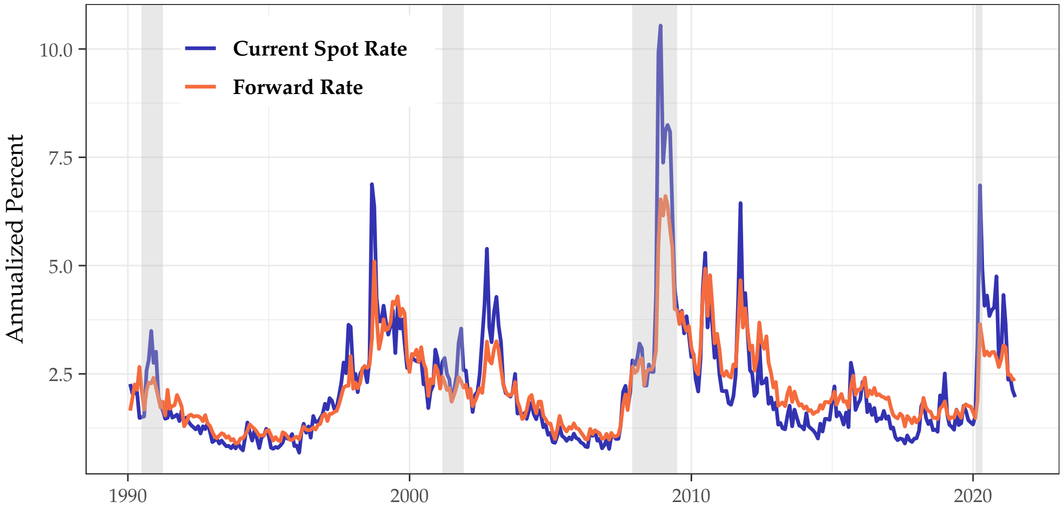
- Global panel of index options from OptionMetrics
  - For U.S. sample: 1990–2021
  - For international sample: Consider 10 major indices, with data since at least 2006
- Sample monthly and apply standard filters
- Baseline: 6-month horizon, 6 months forward ($n = m = 6$)

Measuring LVIX: Following Gao & Martin (2021), Carr & Madan (2001),

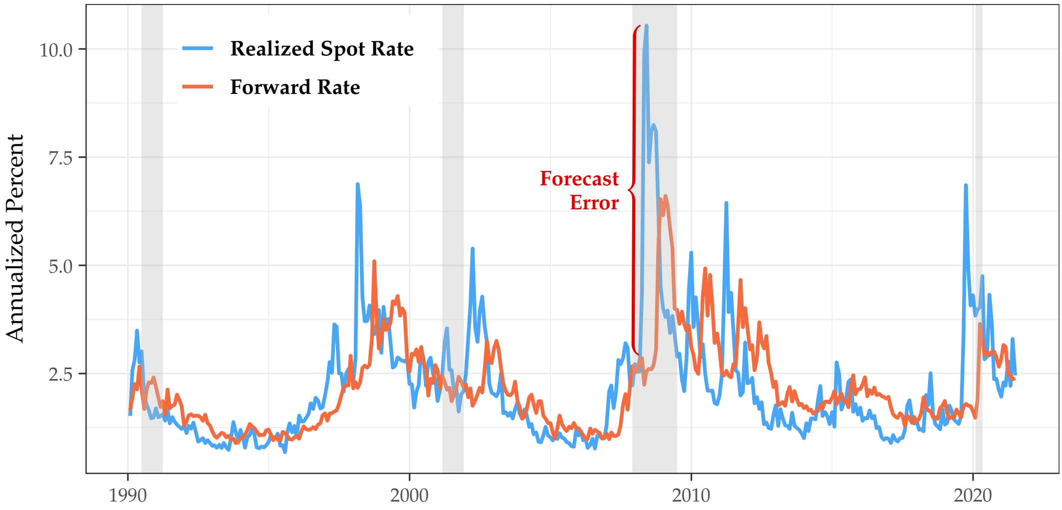
$$\begin{aligned}\mathcal{L}_t^{(n)} &= \left(R^f_{t,t+n}\right)^{-1} \mathbb{E}^*_t\left[R_{t,t+n} r_{t,t+n}\right] - r^f_{t,t+n} \\ &= \frac{1}{P_t}\left\{\int_0^{F_t^{(n)}} \frac{\text{put}_t^{(n)}(K)}{K} dK + \int_{F_t^{(n)}}^{\infty} \frac{\text{call}_t^{(n)}(K)}{K} dK\right\}\end{aligned}$$

- Calculate integral a bunch of different ways (appendix has details)
- First: Simplify by working under log assumption, so LVIX $\Longrightarrow$ spot & forward rates

Estimates: Contemporaneous U.S. Spot and Forward Rates



Estimates: Realized U.S. Spot and Forward Rates



Do Forward Rates Predict Future Spot Rates?

Mincer–Zarnowitz Regressions for Spot Rates by Country

$$\mu_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(6,6)} + \epsilon_{t+6}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(6,6)}$	0.67***	0.55***	0.56***
	(0.096)	(0.056)	(0.055)
Intercept	0.74***		
	(0.28)		
Country FEs	✗	✓	✓
p-value: $\beta_1 = 1$	0.003	0.000	0.000
Obs.	378	1,849	2,227
R^2	0.22	0.21	0.22
Within R^2	—	0.14	0.15

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Substantial predictive power...
- ...but $\beta_1 \neq 1$, suggesting forward rates overshoot future spot rates
- What if β_1 estimate is downwardly biased due to measurement error?
- To address this, now consider IV using shorter-term forward rate $f_t^{(2,1)}$ as instrument for $f_t^{(6,6)}$
- Shorter-horizon forwards likely to be better measured: denser option strikes & more trading volume

Do Forward Rates Predict Future Spot Rates?

Instrumented Mincer–Zarnowitz Regressions for Spot Rates

$$\mu_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(6,6)} + \epsilon_{t+6}, \quad f_t^{(6,6)} = \pi_0 + \pi_1 f_t^{(2,1)} + \eta_t$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(6,6)}$	0.73***	0.69***	0.70***
	(0.062)	(0.078)	(0.074)
Intercept	0.59***		
	(0.13)		
Country FEs	✗	✓	✓
p-value: $\beta_1 = 1$	0.018	0.004	0.003
Obs.	378	1,849	2,227
R^2	0.22	0.20	0.22
Within R^2	—	0.13	0.14

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Forward rate $\nearrow$ by **1%** $\Longrightarrow$ future spot rate $\nearrow$ by $\sim$**0.7%**
- Forward rates explain $\sim$**20%** of the variation in future spot rates
- Now soften log utility assumption and turn to forecast errors

Average Forecast Errors Are Close to Zero

Average Forecast Errors Across Countries

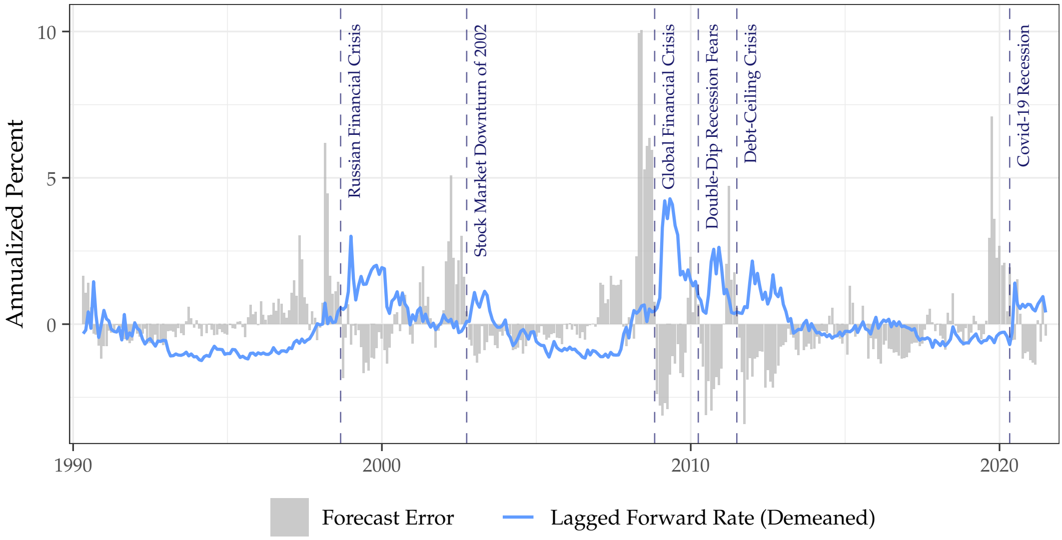
$$\varepsilon^{(6)}_{t+6} = \mu^{(6)}_{t+6} - f^{(6,6)}_t$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
Average	0.021	0.20	0.17
	(0.15)	(0.11)	(0.11)
Obs.	378	1,849	2,227

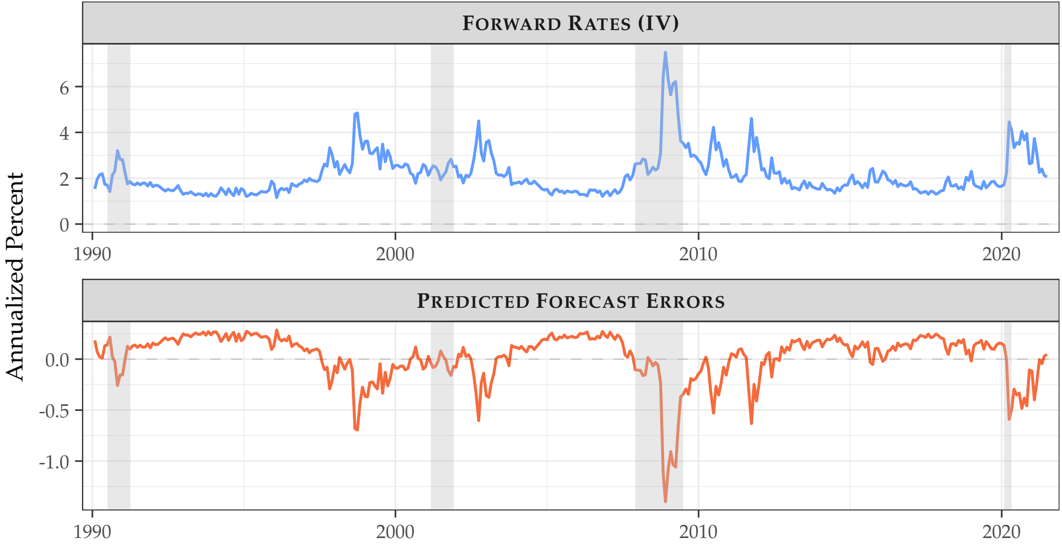
SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Not just statistically insignificant, but effectively zero: $\bar{\varepsilon} \leqslant 20$ bps annualized
- Therefore can't reject log utility + RE just on the basis of average errors
  - Not the highest-powered test, but will be informative in trying to rationalize time variation
- But average of zero masks substantial predictability. . .

Forecast Errors and Lagged Forward Rates Over Time



Forward Rates as Predictors of Forecast Errors



Predictable Forecast Errors

Regressions of Forecast Errors on 2×1 Forward Rate

$$\varepsilon_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(2,1)} + e_{t+6}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(2,1)}$	-0.17**	-0.16**	-0.16***
	(0.066)	(0.049)	(0.047)
Intercept	0.39*		
	(0.23)		
Country FEs	✗	✓	✓
Obs.	378	1,849	2,227
R^2	0.04	0.04	0.04
Within R^2	—	0.03	0.03

SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

Predictable Forecast Errors

Regressions of Forecast Errors on 2×1 Forward Rate

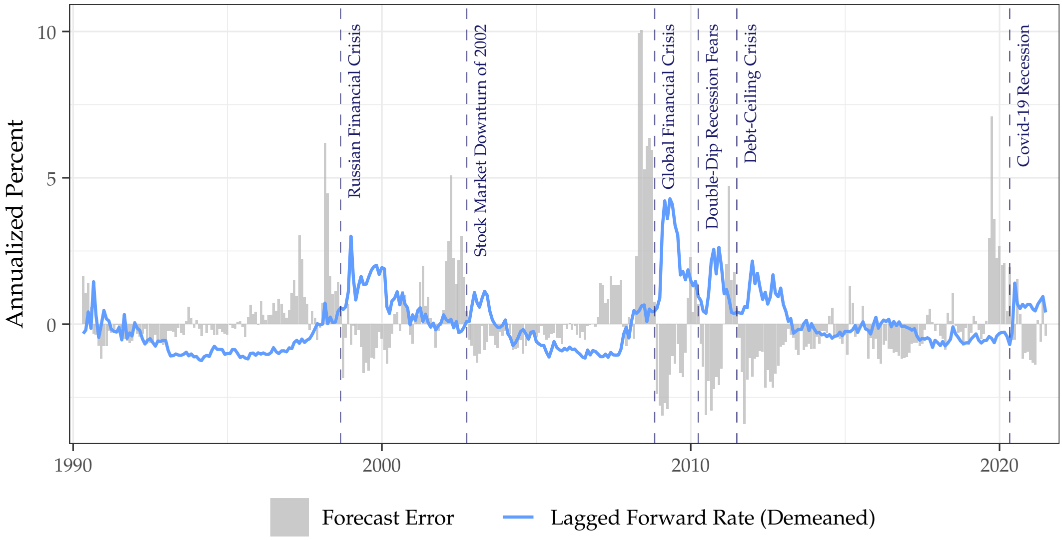
$$\varepsilon_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(2,1)} + e_{t+6}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(2,1)}$	-0.17**	-0.16**	-0.16***
	(0.066)	(0.049)	(0.047)
Intercept	0.39*		
	(0.23)		
Country FEs	✗	✓	✓
Obs.	378	1,849	2,227
R^2	0.04	0.04	0.04
Within R^2	—	0.03	0.03

SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Forward rates again overshoot future spot rates
- Errors are also predictable in Coibion–Gorodnichenko regressions using forward-rate *revisions* (more shortly)
- And predictability rises substantially ($R^2 = 0.11$) with kernel regression: Arises mostly from high forward rates
- Is this consistent with "overreaction"? It depends: Overreaction to what?
  - Option-based expected returns: Yes
    [Spot rates, fwd rates, fwd-rate revisions]
  - Past returns: Wrong direction!

Reminder: Forecast Errors and Lagged Forward Rates



How Significant Are Forecast Errors?

Before taking any stand on source of estimated forecast errors *[expectation errors vs. risk premia]*, return to question posed at outset: **How significant are they for price variation?**

$$p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

How Significant Are Forecast Errors?

Before taking any stand on source of estimated forecast errors *[expectation errors vs. risk premia]*, return to question posed at outset: **How significant are they for price variation?**

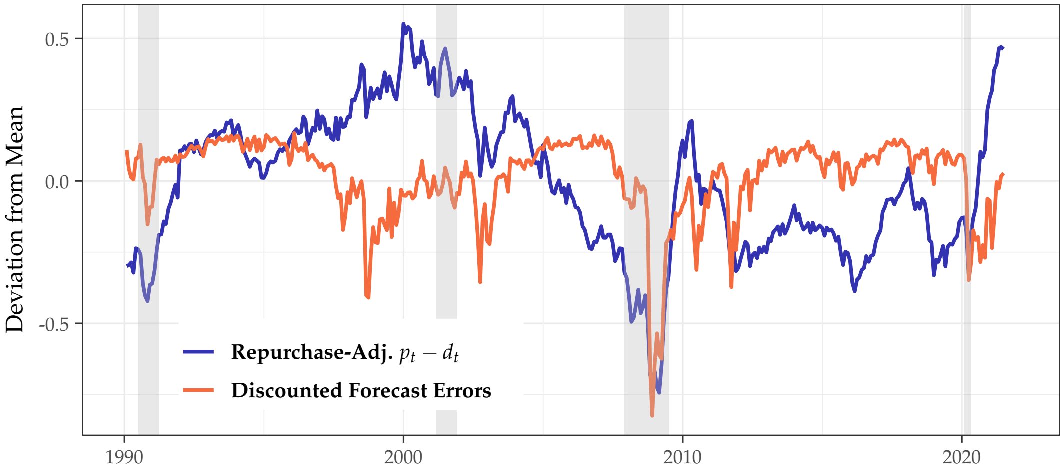
$$p_t - d_t \;=\; \kappa \;-\; \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f_t^{(j,1)} - \underbrace{RF_t}_{\substack{\text{discounted} \\ \text{risk-free rates}}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

Break $f_t^{(j,1)}$ into:

$$f_t^{(j,1)} = \underbrace{\mathbb{E}_t[\mu_{t+j}^{(1)}]}_{\text{expected spot rates}} + \underbrace{\mathbb{E}_t[\varepsilon_{t+j}^{(1)}]}_{\text{predictable forecast errors}}$$

- Set one period to be 3 months, and predict $f_{t+j}^{(1)}$ for $j = 2, 3$ (6 & 9 months ahead) using 2m×1m forward
- Assume $\mathbb{E}_t[\varepsilon_{t+j+1}^{(1)}] = \phi^j \mathbb{E}_t[\varepsilon_{t+j}^{(1)}]$ [De la O & Myers (2021)] $\Longrightarrow \widehat{\phi} \approx 1$ *(also holds using longer-dated SX5E data)*
- Use this to calculate discounted sum of predicted forecast errors' maximal possible effect on prices
- Compare to repurchase-adj. $p_t - d_t$ from Nagel & Xu (2022)

Discounted Forecast Errors and Price-Dividend Variation



Meaningful in magnitude, esp. during crisis, and overall accounts for 8% of $p_t - d_t$ variation

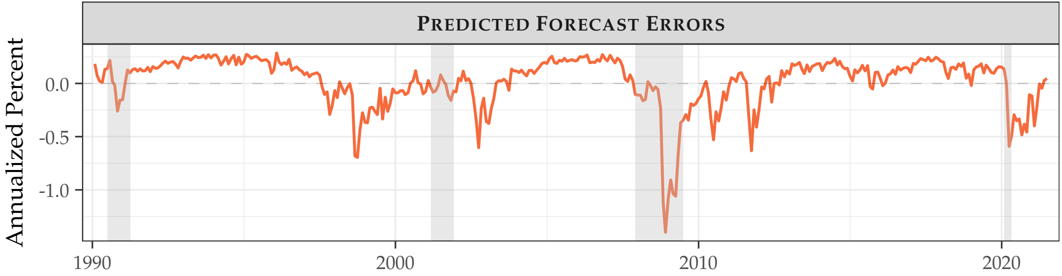
Roadmap

1. Introduction
2. Theory
3. Implementation and Results
4. Rationalizing Forecast Errors
5. A Model of Expectation Errors
6. Discussion and Conclusions

Can Forecast Errors Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \mu^{(m)}_{t+n}\right)}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?
Must have $-\varsigma_t$ take same sign as pred. forecast errors:



Main challenge: Small on average, but must flip signs dramatically (− in good times, + in bad).

Can Forecast Errors Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \mu^{(m)}_{t+n}\right)}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?

- For simplicity: Take $n = m = 1$, and assume M_{t+1}, R_{t+1}, μ_{t+1} jointly log-normal
- Then $\varsigma_t > 0$ (as needed in bad times) if and only if:

  $$SR_t(-\mu_{t+1}) > -\rho_t(r,\mu)\sigma_t(r),$$

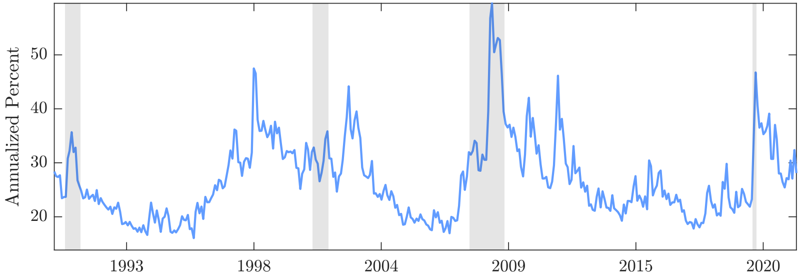
  where SR_t is Sharpe ratio on claim to next period's negative equity premium (low payoff is bad)
- Correlation $\rho_t(r,\mu)$ likely to be negative; for illustration, set it to -1
- Then SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- One calibration: Go back to log utility (likely to be conservative for time variation in σ_t), and estimate $\sigma_t(r)$ from options

Can Forecast Errors Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \mu^{(m)}_{t+n}\right)}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?

- SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- One calibration: Go back to conservative log utility case, and estimate $\sigma_t(r)$ from options. **Results for conditional volatility of 6-month market return:**



Can Forecast Errors Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \underbrace{\text{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \mu_{t+n}^{(m)}\right)}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to be unpredictable?

- SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- Further, given $\rho_t(r,\mu) = -1$, ς_t **cannot** flip signs if mNCC [Gao & Martin (2021), Assumption 2] holds
  - $\rho_t(r,\mu) = -1 \implies \varsigma_t$ is scaled version of their covariance term $\mathcal{C}_t^{(n)}$
  - If $\mathcal{C}_t^{(n)} \leqslant 0$ (mNCC), then $\varsigma_t \leqslant 0$
- More generally, difficult to get both average errors (small) *and* time variation (large) right
- Paper has one illustration varying risk aversion γ

Roadmap

1. Introduction
2. Theory
3. Implementation and Results
4. Rationalizing Forecast Errors
5. A Model of Expectation Errors
6. Discussion and Conclusions

Model Setup

- Now want a simple lab to examine whether findings could plausibly arise from combo of:
  1. Log utility
  2. Expectation errors

⇒ consider a version of framework in Bordalo, Gennaioli, Shleifer (2018), Augenblick & Rabin (2021)

- 3-month spot rate dynamics under **objective** measure:

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1\, \mu_{t-1}^{(3)} + \phi_2\, \mu_{t-2}^{(3)} + \phi_3\, \mu_{t-3}^{(3)} + \epsilon_t, \qquad \epsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$$

- **Under RE:** Term structure of current spot rates would be based on objective expectations $\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right]$
- **Actual subjective expectations:** Excess sensitivity to news governed by "diagnosticity" parameter θ:

$$\mathbb{E}_t^{\theta}\left[\mu_{t+n}^{(3)}\right] = \mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] + \theta \underbrace{\left(\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] - \mathbb{E}_{t-3}\left[\mu_{t+n}^{(3)}\right]\right)}_{\text{news} \propto \epsilon_t}$$

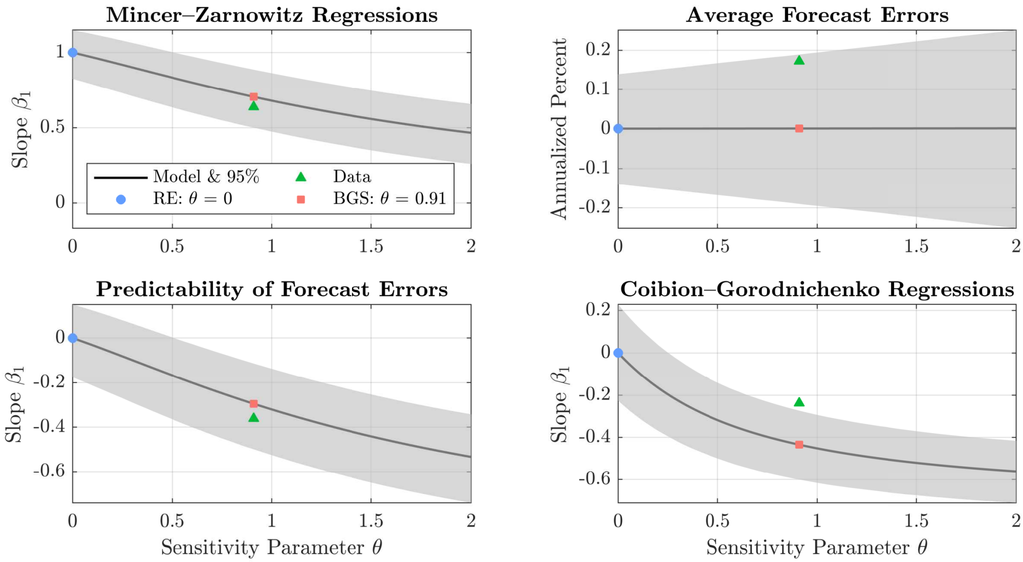
Model Setup

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1\, \mu_{t-1}^{(3)} + \phi_2\, \mu_{t-2}^{(3)} + \phi_3\, \mu_{t-3}^{(3)} + \epsilon_t, \qquad \epsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$$

$$\mathbb{E}_t^{\theta}\left[\mu_{t+n}^{(3)}\right] = \mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] + \theta \underbrace{\left(\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] - \mathbb{E}_{t-3}\left[\mu_{t+n}^{(3)}\right]\right)}_{\text{news} \propto \epsilon_t}$$

- Forward rates based on subjective expectations
- Longer-term spot rates embed objective short rate **and** subjective expectations of future short rates
- Consider a range of values for θ
  - $\theta = 0$: RE
  - $\theta = 0.91$: BGS (2018)
- Estimate objective parameters for spot-rate process in each country
- For each θ, simulate 10,000 samples and run same tests as in the data for $n, m = 6$ months

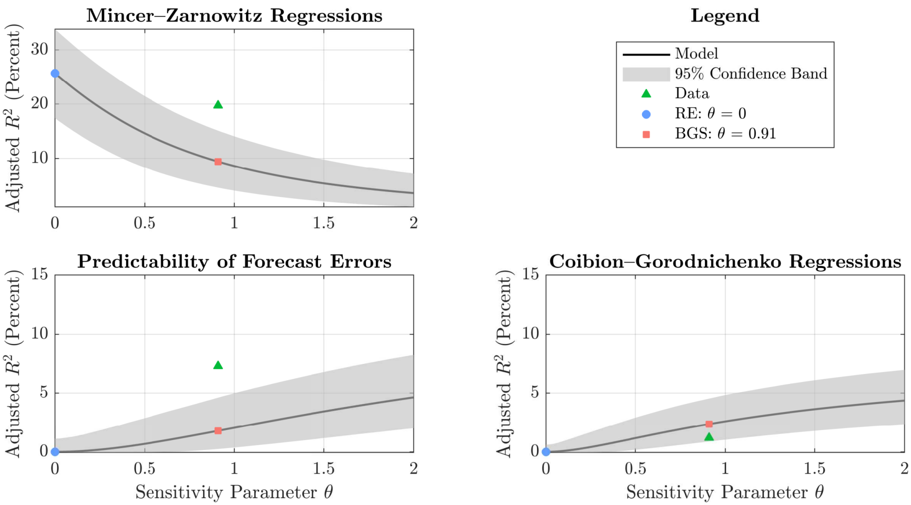
Model vs. Data: Main Estimates



Model vs. Data: R^2 Values

Simple calibration does reasonably well on main estimates…

…**but seems to miss some rational variation in forward rates:**



A Trilemma for Expectation Errors

More generally:

- While simple calibrated model does reasonably well at matching the data, again not an unqualified success for all possible notions of overreaction
- Subjective beliefs overreact to increases in *spot rates* in our model, not past returns, and cyclicality matters:

$$p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1} \mathbb{E}_t r_{t+1} - \underbrace{\sum_{j=1}^{\infty} \rho^j f_t^{(j,1)}}_{\mathcal{F}_t} + \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}}_{CF_t} - RF_t$$

- Use $\widetilde{\cdot}$ to denote **expectation error wedge** (deviation from RE economy):

$$\text{var}\left(\widetilde{p_t - d_t}\right) = \text{var}\left(\widetilde{\mathcal{F}}_t\right) + \text{var}\left(\widetilde{CF}_t\right) - 2\,\text{cov}\left(\widetilde{\mathcal{F}}_t, \widetilde{CF}_t\right)$$

- Have to choose between **two of three**:
  1. Volatile expectation errors for cash flows and/or returns
  2. Volatile price-dividend ratio relative to RE
  3. Positive comovement between fundamental and return expectation errors

Roadmap

1. Introduction
2. Theory
3. Implementation and Results
4. Rationalizing Forecast Errors
5. A Model of Expectation Errors
6. Discussion and Conclusions

Final Notes

Summary:

- Introduce new methodology to test whether the market understands time variation in equity premium
- Find evidence that it does. . .to an extent

Tie-ins:

- Equity and fixed-income term structure
- Our tests are similar to tests of the expectations hypothesis, but with less room for discount-rate variation than in previous versions
- Similar to past work [van Binsbergen & Koijen (2017), Gormsen (2021)], find more predictability in equity term structure than in FI term structure
- Also build on Giglio & Kelly (2018) work on other term structures

Still to do: Additional tests, more work on potential rational discount-rate variation, . . .