Does the Market Understand Time Variation in the Equity Premium?

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Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?
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\[ p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} \]

\[ \mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} \]

much more important for pricing!
Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
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\[ p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} \]

Our focus:

- What is the expected future equity premium?
- How does it compare to the actual future equity premium \( \mathbb{E}_{t+j} r_{t+j+1} \)?
- Are there systematic errors in expected return predictions?
What We Do

1. Option-based measure of log equity premium:

   **Spot rate:** \[ \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n]} - r_{t,t+n}^f \]

2. Calculate expected future equity premium:

   **Forward rate:** \[ f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \]

3. Compare forward rate to realized future spot rate:

   **Forecast error:** \[ \epsilon_{t+n} = \mu_{t+n} - f_t^{(n)} \]
Why This Framework?

Spot rate: \[ \mu_t^{(n)} = E_t[r_{t,t+n}] - r_{t,t+n}^f \]

Forward rate: \[ f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = E_t[\mu_{t+n}^{(1)}] \]

Forecast error: \[ \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)} \]

- Term structure of log equity premia \( \implies \) straightforward calculation of forward rates
- Behavior of term structure:
  - Key for prices
  - Great lab for testing whether market-based expectations are intertemporally consistent, without needing to take a stand on whether expected returns are themselves rational
- Forecast errors require much weaker conditions than needed to estimate \( \mu_t^{(n)} \) and \( f_t^{(n,m)} \) separately
  - In general, spot and forward rates are contaminated by SDF-related covariances . . .
  - . . .but when differencing, the covariance terms largely cancel
What We Find

Spot rate: \[ \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}] - r_{t,t+n}^f \]

Forward rate: \[ f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \]

Forecast error: \[ \epsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)} \]

In global options sample:

1. Forward rates are strong predictors of realized spot rates
   - Forward rate \( \uparrow \) by 1% \( \implies \) future spot rate \( \uparrow \) by 0.7%
   - Forward rates explain 20% of the variation in future spot rates at 6-month horizon

2. Forecast errors are close to 0 on average...

3. ...but still exhibit predictable mean reversion and excess volatility
   - Can therefore reject fully rational expectations for range of possible SDF specifications
   - Alternatively, need highly volatile and countercyclical price of discount-rate risk
Illustration: U.S. Forward and Realized Spot Rates in Three Crises

- **1998 Russian Debt Crisis**
- **2008 Financial Crisis**
- **2020 COVID-19 Recession**

- **Forward rate at crisis onset**
- **Realized one-month spot rate**
Roadmap

1. Introduction

2. Theory
   - The Log Utility Case
   - The General Case

3. Implementation and Results

4. Rationalizing Forecast Errors

5. A Model of Expectation Errors

6. Discussion and Conclusions
The Log Utility Case: Setup

Setting:

- Representative agent (“the market”)
  - Equivalently, any unconstrained investor who chooses to fully invest in the market
- For now: Log utility over wealth

\[
\log(W_{t+n}) = \log(W_t R_{t,t+n})
\]

\[
P_t = \mathbb{E}_t[M_{t,t+n} \text{Payoff}_{t+n}]
\]

\[
M_{t,t+n} = 1/R_{t,t+n}
\]

- Useful for exposition, but will turn out not to be central
The Log Utility Case: Setup

Setting:

- Representative agent (“the market”)
  - Equivalently, any unconstrained investor who chooses to fully invest in the market
- For now: Log utility over wealth $\Leftrightarrow M_{t,t+n} = 1/R_{t,t+n}$
- Building block: LVIX $\mathcal{L}_t^{(n)}$ [Gao and Martin (2021)]:
  \[
  \mathbb{E}_t[r_{t,t+n}] - r_{f,t,t+n}^{\mu_t(n)} = (R_{t,t+n}^{f})^{-1} \mathbb{E}_t^*[R_{t,t+n}r_{t,t+n}] - r_{f,t,t+n}^{\mu_t(n)} - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})
  \]
- $\mathcal{L}_t^{(n)}$: Observable from options (details later)
- Second term: $MR = 1 \implies \text{Cov} = 0$ under log utility
The Log Utility Case: Result

General notation:

Spot rate: \[ \mu^{(n)}_t = \mathbb{E}_t[r_{t,t+n}] - r^{f}_{t,t+n} \]

Forward rate: \[ f^{(n,m)}_t = \mu^{(n+m)}_t - \mu^{(n)}_t = \mathbb{E}_t[\mu^{(m)}_{t+n}] \]

Forecast error: \[ \varepsilon^{(m)}_{t+n} = \mu^{(m)}_{t+n} - f^{(n,m)}_t \]

Timeline:
The Log Utility Case: Result

General notation:

Spot rate: \( \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}] - f_{t,t+n} \)

Forward rate: \( f_t^{(n,m)} = \mu_t^{(n+m)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(m)}] \)

Forecast error: \( \varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)} \)

Result 1 (Log Utility Identification)

Given \( M_{t,t+n} = 1/R_{t,t+n} \):

\[
\begin{align*}
\mu_t^{(n)} &= \mathcal{L}_t^{(n)} \\
\mu_t^{(n)} &= \mathcal{L}_t^{(n)} \\
f_t^{(n,m)} &= \mathcal{L}_t^{(n+m)} - \mathcal{L}_t^{(n)} \\
\varepsilon_{t+n}^{(m)} &= \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_t^{(n+m)} + \mathcal{L}_t^{(n)}
\end{align*}
\]

▫ Can then examine term structure, test \( \mathbb{E}[\varepsilon_{t+n}^{(m)}] = 0, \mathbb{E}[Z_t\varepsilon_{t+n}^{(m)}] = 0, \ldots \)
The General Case: Identification Challenge

Beyond log utility?

- In general, spot and forward rates are contaminated by unobservable covariances:

\[
\mu_{t}^{(n)} = \mathcal{L}_{t}^{(n)} - \text{Cov}_{t}(M_{t,t+n}R_{t,t+n}, r_{t,t+n})
\]

\[
f_{t}^{(n,m)} = \mathcal{L}_{t}^{(n+m)} - \mathcal{L}_{t}^{(n)} + C_{t}^{(n)} - C_{t}^{(n+m)}
\]

- For \(\mu_{t}^{(n)}\), can argue \(C_{t}^{(n)} \leq 0\) [Gao & Martin (2021)]. .. but for \(f_{t}^{(n,m)}\), \(C_{t}^{(n+m)} \gg C_{t}^{(n)}\)

- Key insight: Covariance terms largely cancel when considering forecast errors \(\epsilon_{t+n}^{(m)} = \mu_{t+n} - f_{t}^{(n,m)}\)

- Under log-normality, for \(n = m = 1\):

\[
\mathbb{E}_{t}[C_{t+1}^{(1)}] + C_{t}^{(1)} - C_{t}^{(2)} = \text{Cov}_{t}(M_{t,t+1}R_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]) = \text{much less volatile than } r_{t+1,t+2}
\]
The Log-Normal Case: Result

Define forecast-error proxy from log utility case:

$$\hat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t}^{(n+m)} + \mathcal{L}_{t}^{(n)}$$

To continue building toward general case…

Result 2 (Log-Normal Identification)

For a general SDF $M_{t,t+n}$, assuming $M_{t,t+n}$, $R_{t,t+n}$ are jointly log-normal:

$$\mathbb{E}_t \left[ \hat{\varepsilon}_{t+n}^{(m)} \right] = \mathbb{E}_t \left[ \varepsilon_{t+n}^{(m)} \right] - \text{Cov}_t (M_{t,t+n}R_{t,t+n}, \mathbb{E}_{t+n} [r_{t+n,t+n+m}])$$

- Covariance term now relates to pricing of discount-rate risk, rather than realized-return risk
- Basic idea of proof: $MR_{t,t+n}$ is orthogonal to unexpected component of $r_{t+n,t+n+m}$, so left with expectation term $\mathbb{E}_{t+n} [r_{t+n,t+n+m}]$
- Remaining covariance is likely quite small, but can be disciplined empirically or theoretically
The General Case: Result

Define forecast-error proxy and expected-return proxy:

\[ \hat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t+n}^{(n+m)} + \mathcal{L}_{t}^{(n)} \]

\[ \hat{\mu}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} + r_{t+n,t+n+m}^{f} \]

**Result 3 (Generalized Identification)**

For any SDF \( M_{t,t+n} \),

\[ \mathbb{E}_{t} \left[ \hat{\varepsilon}_{t+n}^{(m)} \right] = \mathbb{E}_{t} \left[ \varepsilon_{t+n}^{(m)} \right] - \text{Cov}_{t} \left( M_{t,t+n} R_{t,t+n}, \hat{\mu}_{t+n}^{(m)} \right) \]

- Intuition from log-normal case carries over, with \( \mathbb{E}_{t+n} \left[ r_{t+n,t+n+m} \right] \) replaced by \( \hat{\mu}_{t+n}^{(m)} \)
- LVIX-based \( \hat{\mu}_{t+n}^{(m)} \) is closely related to \( \mathbb{E}_{t+n} \left[ r_{t+n,t+n+m} \right] \)…but \( \hat{\mu}_{t+n}^{(m)} \) is directly observable
- Main specification: \( \hat{\mu}_{t+n}^{(m)} \) is \( \frac{1}{10} \) as volatile as realized return \( r_{t+n,t+n+m} \)
  \[ \implies \] unobserved covariance likely much smaller for forecast errors than for spot rates
Roadmap

1. Introduction

2. Theory

3. Implementation and Results
   - Data
   - Main Estimates
   - Forecast Errors and Predictability
   - Interpretation

4. Rationalizing Forecast Errors

5. A Model of Expectation Errors

6. Discussion and Conclusions
Data and Measurement

Data:
- Global panel of index options from OptionMetrics
  - For U.S. sample: 1990–2021
  - For international sample: Consider 10 major indices, with data since at least 2006
- Sample monthly and apply standard filters
- Baseline: 6-month horizon, 6 months forward ($n = m = 6$)

Measuring LVIX: Following Gao & Martin (2021), Carr & Madan (2001),

$$
\mathcal{L}^{(n)}_t = \mathbb{E}^*_{t} [R_{t,t+n} r_{t,t+n}] - r^{f}_{t,t+n}
$$

$$
= \frac{1}{P_t} \left\{ \int_{0}^{F_i^{(n)}} \frac{\text{put}_i^{(n)}(K)}{K} dK + \int_{F_i^{(n)}}^{\infty} \frac{\text{call}_i^{(n)}(K)}{K} dK \right\}
$$

- Calculate integral a bunch of different ways (appendix has details)
- First: Simplify by working under log assumption, so LVIX $\implies$ spot & forward rates
Estimates: Contemporaneous U.S. Spot and Forward Rates

Graph showing annualized percent for Current Spot Rate and Forward Rate from 1990 to 2020.
Estimates: **Realized U.S. Spot and Forward Rates**

- Annualized Percent
- Realized Spot Rate
- Forward Rate

Forecast Error
### Mincer–Zarnowitz Regressions for Spot Rates by Country

\[
\mu_{t+6} = \beta_0 + \beta_1 f^{(6,6)}_t + \epsilon_{t+6}
\]

<table>
<thead>
<tr>
<th></th>
<th>(1) U.S.</th>
<th>(2) Ex-U.S.</th>
<th>(3) All</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f^{(6,6)}_t)</td>
<td>0.67***</td>
<td>0.55***</td>
<td>0.56***</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.056)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.74***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country FEs</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>(p)-value: (\beta_1 = 1)</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Obs.</td>
<td>378</td>
<td>1,849</td>
<td>2,227</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.22</td>
<td>0.21</td>
<td>0.22</td>
</tr>
<tr>
<td>Within (R^2)</td>
<td>—</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Substantial predictive power...
- ...but \(\beta_1 \neq 1\), suggesting forward rates overshoot future spot rates
- What if \(\beta_1\) estimate is downwardly biased due to measurement error?
- To address this, now consider IV using shorter-term forward rate \(f^{(2,1)}_t\) as instrument for \(f^{(6,6)}_t\)
- Shorter-horizon forwards likely to be better measured: denser option strikes & more trading volume
### Instrumented Mincer–Zarnowitz Regressions for Spot Rates

\[
\mu_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(6,6)} + \epsilon_{t+6}, \quad f_t^{(6,6)} = \pi_0 + \pi_1 f_t^{(2,1)} + \eta_t
\]

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<td>(f_t^{(6,6)})</td>
<td>0.73***</td>
<td>0.69***</td>
<td>0.70***</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.078)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.59***</td>
<td></td>
<td></td>
</tr>
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<td></td>
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<td>✓</td>
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<tr>
<td>p-value: (\beta_1 = 1)</td>
<td>0.018</td>
<td>0.004</td>
<td>0.003</td>
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- Forward rate \(\uparrow\) by 1% \(\implies\) future spot rate \(\uparrow\) by \(~0.7\%\)
- Forward rates explain \(~20\%) of the variation in future spot rates
- Now soften log utility assumption and turn to forecast errors

*SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.*
Average Forecast Errors Are Close to Zero

Average Forecast Errors Across Countries

\[ \varepsilon_{t+6}^{(6)} = \mu_{t+6}^{(6)} - f_t^{(6,6)} \]

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<td>Average</td>
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<td>0.20</td>
<td>0.17</td>
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<td>(0.15)</td>
<td>(0.11)</td>
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SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

▶ Not just statistically insignificant, but effectively zero: \( \bar{\varepsilon} \leq 20 \) bps annuallized
▶ Therefore can’t reject log utility + RE just on the basis of average errors
  ▶ Not the highest-powered test, but will be informative in trying to rationalize time variation
▶ But average of zero masks substantial predictability…
Forward Rates as Predictors of Forecast Errors

**Forward Rates (IV)**

**Predicted Forecast Errors**
Predictable Forecast Errors

Regressions of Forecast Errors on $2 \times 1$ Forward Rate

$$\varepsilon^{(6)}_{t+6} = \beta_0 + \beta_1 f^{(2,1)}_t + e_{t+6}$$

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Country FEs | X | ✓ | ✓ |

Obs. | 378 | 1,849 | 2,227 |
$R^2$ | 0.04 | 0.04 | 0.04 |
Within $R^2$ | — | 0.03 | 0.03 |

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Predictable Forecast Errors

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- Forward rates again overshoot future spot rates
- Errors are also predictable in Coibion–Gorodnichenko regressions using forward-rate revisions (more shortly)
- And predictability rises substantially ($R^2 = 0.11$) with kernel regression: Arises mostly from high forward rates
- Is this consistent with “overreaction”? It depends: Overreaction to what?
- Option-based expected returns: Yes [Spot rates, fwd rates, fwd-rate revisions]
- Past returns: Wrong direction!
Reminder: Forecast Errors and Lagged Forward Rates

Annualized Percent

Forecast Error  Lagged Forward Rate (Demeaned)
How Significant Are Forecast Errors?

Before taking any stand on source of estimated forecast errors [expectation errors vs. risk premia], return to question posed at outset: How significant are they for price variation?

\[ p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} \]
How Significant Are Forecast Errors?

Before taking any stand on source of estimated forecast errors \([\text{expectation errors vs. risk premia}]\), return to question posed at outset: How significant are they for price variation?

\[
p_t - d_t = \kappa - \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f^{(j,1)}_t - RF_t + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1}
\]

Break \( f^{(j,1)}_t \) into:

\[
f^{(j,1)}_t = \mathbb{E}_t [\mu^{(1)}_{t+j}] + \mathbb{E}_t [\epsilon^{(1)}_{t+j}]
\]

- Expected spot rates
- Predictable forecast errors

▶ Set one period to be 3 months, and predict \( f^{(1)}_{t+j} \) for \( j = 2, 3 \) (6 & 9 months ahead) using 2m × 1m forward

▶ Assume \( \mathbb{E}_t [\epsilon^{(1)}_{t+j+1}] = \phi^j \mathbb{E}_t [\epsilon^{(1)}_{t+j}] \) \([\text{De la O & Myers (2021)}]\) \( \implies \hat{\phi} \approx 1 \) (also holds using longer-dated SX5E data)

▶ Use this to calculate discounted sum of predicted forecast errors’ maximal possible effect on prices

▶ Compare to repurchase-adj. \( p_t - d_t \) from Nagel & Xu (2022)
Discounted Forecast Errors and Price-Dividend Variation

Meaningful in magnitude, esp. during crisis, and overall accounts for 8% of $p_t - d_t$ variation
Roadmap

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2. Theory
3. Implementation and Results
4. Rationalizing Forecast Errors
5. A Model of Expectation Errors
6. Discussion and Conclusions
Can Forecast Errors Be Rationalized?

\[
E_t[\hat{\varepsilon}_{t+n}^{(m)}] = E_t[\varepsilon_{t+n}^{(m)}] - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, \mu_{t+n}^{(m)})
\]

What conditions do we need on \( \zeta_t \) in order for expectation errors \( E_t[\varepsilon_{t+n}^{(m)}] \) to be unpredictable?

Must have \(-\zeta_t\) take same sign as pred. forecast errors:

Main challenge: Small on average, but must flip signs dramatically (− in good times, + in bad).
Can Forecast Errors Be Rationalized?

\[ \mathbb{E}_t[\hat{\epsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\epsilon^{(m)}_{t+n}] - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, \mu^{(m)}_{t+n}) \]

What conditions do we need on \( \varsigma_t \) in order for expectation errors \( \mathbb{E}_t[\epsilon^{(m)}_{t+n}] \) to be unpredictable?

- For simplicity: Take \( n = m = 1 \), and assume \( M_{t+1}, R_{t+1}, \mu_{t+1} \) jointly log-normal.
- Then \( \varsigma_t > 0 \) (as needed in bad times) if and only if:

\[ SR_t(-\mu_{t+1}) > -\rho_t(r, \mu)\sigma_t(r), \]

where \( SR_t \) is Sharpe ratio on claim to next period’s negative equity premium (low payoff is bad).

- Correlation \( \rho_t(r, \mu) \) likely to be negative; for illustration, set it to \(-1\).
- Then \( SR_t \) must vary more than \( \sigma_t(r) \) for \( \varsigma_t \) to flip signs.
- One calibration: Go back to log utility (likely to be conservative for time variation in \( \sigma_t \)), and estimate \( \sigma_t(r) \) from options.
Can Forecast Errors Be Rationalized?

\[ \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, \mu_{t+n}^{(m)}) \]

What conditions do we need on \( \varsigma_t \) in order for expectation errors \( \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] \) to be unpredictable?

- \( SR_t \) must vary more than \( \sigma_t(r) \) for \( \varsigma_t \) to flip signs

- One calibration: Go back to conservative log utility case, and estimate \( \sigma_t(r) \) from options.

Results for conditional volatility of 6-month market return:
Can Forecast Errors Be Rationalized?

\[
E_t[\hat{\varepsilon}_{t+n}^{(m)}] = E_t[\varepsilon_{t+n}^{(m)}] - \text{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \mu_{t+n}^{(m)}\right)_{\xi_t}
\]

What conditions do we need on \(\varsigma_t\) in order for \textbf{expectation errors} \(E_t[\varepsilon_{t+n}^{(m)}]\) to be unpredictable?

\begin{itemize}
  \item \(SR_t\) must vary \textit{more than} \(\sigma_t(r)\) for \(\varsigma_t\) to flip signs
  \item Further, given \(\rho_t(r, \mu) = -1\), \(\varsigma_t\) \textbf{cannot} flip signs if mNCC \cite[Gao & Martin (2021), Assumption 2] holds
    \begin{itemize}
      \item \(\rho_t(r, \mu) = -1 \implies \varsigma_t\) is scaled version of their covariance term \(C_t^{(n)}\)
      \item If \(C_t^{(n)} \leq 0\) (mNCC), then \(\varsigma_t \leq 0\)
    \end{itemize}
  \item More generally, difficult to get both average errors (small) \textit{and} time variation (large) right
  \item Paper has one illustration varying risk aversion \(\gamma\)
\end{itemize}
Roadmap

1. Introduction
2. Theory
3. Implementation and Results
4. Rationalizing Forecast Errors
5. A Model of Expectation Errors
6. Discussion and Conclusions
Model Setup

Now want a simple lab to examine whether findings could plausibly arise from combo of:

1. Log utility
2. Expectation errors

Consider a version of framework in Bordalo, Gennaioli, Shleifer (2018), Augenblick & Rabin (2021)

3-month spot rate dynamics under objective measure:

\[
\mu^{(3)}_t = \left(1 - \sum_{j=1}^{3} \phi_j \right) \bar{\mu} + \phi_1 \mu^{(3)}_{t-1} + \phi_2 \mu^{(3)}_{t-2} + \phi_3 \mu^{(3)}_{t-3} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_{\epsilon})
\]

Under RE: Term structure of current spot rates would be based on objective expectations \( \mathbb{E}_t \left[\mu^{(3)}_{t+n}\right] \)

Actual subjective expectations: Excess sensitivity to news governed by “diagnosticity” parameter \( \theta \):

\[
\mathbb{E}_t^{\theta} \left[\mu^{(3)}_{t+n}\right] = \mathbb{E}_t \left[\mu^{(3)}_{t+n}\right] + \theta \left( \mathbb{E}_t \left[\mu^{(3)}_{t+n}\right] - \mathbb{E}_{t-3} \left[\mu^{(3)}_{t+n}\right] \right) \quad \text{(news} \propto \epsilon_t)
Model Setup

\[
\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1 \mu_{t-1}^{(3)} + \phi_2 \mu_{t-2}^{(3)} + \phi_3 \mu_{t-3}^{(3)} + \epsilon_t, \quad \epsilon_t \sim i.i.d. \mathcal{N}(0, \sigma^2_{\epsilon})
\]

\[
E_t^\theta \left[ \mu_{t+n}^{(3)} \right] = E_t \left[ \mu_{t+n}^{(3)} \right] + \theta \left( E_t \left[ \mu_{t+n}^{(3)} \right] - E_{t-3} \left[ \mu_{t+n}^{(3)} \right] \right)
\text{news} \propto \epsilon_t
\]

- Forward rates based on subjective expectations
- Longer-term spot rates embed objective short rate \textbf{and} subjective expectations of future short rates
- Consider a range of values for \( \theta \)
  - \( \theta = 0: \text{RE} \)
  - \( \theta = 0.91: \text{BGS (2018)} \)
- Estimate objective parameters for spot-rate process in each country
- For each \( \theta \), simulate 10,000 samples and run same tests as in the data for \( n, m = 6 \) months
Model vs. Data: Main Estimates

Mincer–Zarnowitz Regressions

Predictability of Forecast Errors

Average Forecast Errors

Coibion–Gorodnichenko Regressions

Slope $\beta_1$

Sensitivity Parameter $\theta$

Slope $\beta_1$

Sensitivity Parameter $\theta$
Model vs. Data: $R^2$ Values

Simple calibration does reasonably well on main estimates...

...but seems to miss some rational variation in forward rates:
A Trilemma for Expectation Errors

More generally:

- While simple calibrated model does reasonably well at matching the data, again not an unqualified success for all possible notions of overreaction

- Subjective beliefs overreact to increases in *spot rates* in our model, not past returns, and cyclicality matters:

\[
p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f_t^{(j,1)} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} - RF_t
\]

- Use \( \sim \) to denote **expectation error wedge** (deviation from RE economy):

\[
\text{var}(p_t - d_t) = \text{var}(\sim F_t) + \text{var}(\sim CF_t) - 2 \text{cov}(\sim F_t, \sim CF_t)
\]

- Have to choose between **two of three**:
  1. Volatile expectation errors for cash flows and/or returns
  2. Volatile price-dividend ratio relative to RE
  3. Positive comovement between fundamental and return expectation errors
Roadmap

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Summary:
▶ Introduce new methodology to test whether the market understands time variation in equity premium
▶ Find evidence that it does...to an extent

Tie-ins:
▶ Equity and fixed-income term structure
▶ Our tests are similar to tests of the expectations hypothesis, but with less room for discount-rate variation than in previous versions
▶ Similar to past work [van Binsbergen & Koijen (2017), Gormsen (2021)], find more predictability in equity term structure than in FI term structure
▶ Also build on Giglio & Kelly (2018) work on other term structures

Still to do: Additional tests, more work on potential rational discount-rate variation, ...