Sentiment and Speculation in a Market with Heterogeneous Beliefs

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Background

Two (overlapping) categories of literature on **belief disagreement**:

1. Heterogeneity + short-sale constraints $\Longrightarrow$ overvaluation
   [Miller (1977), Harrison & Kreps (1978), Scheinkman & Xiong (2003), ...]
   - Useful for explaining speculative bubbles
   - *Not* useful — harmful, in fact — for generating unconditional aggregate equity premium
2. Heterogeneity + borrowing & dynamic trading $\Longrightarrow$ excess trading & volatility
   [Shiller (1984), DeLong, Shleifer, Summers, & Waldmann (1990), David (2008), Banerjee & Kremer (2010), Geanakoplos (2010), Barberis, Greenwood, Jin, & Shleifer (2015), Atmaz & Basak (2018), ...]
   - Rich literature; many different settings & conclusions
   - Papers are either stylized (built to match only a few data features) or technically challenging

This paper lives in the second category. Do we need another paper in that list?

- Yes! Paper provides useful insights: matches features of aggregate data with elegance & simplicity
- Seems to me a very useful minimal dynamic model of heterogeneity with complete markets

Outline

1. Summary: Setting and Results

2. Alternative Interpretations

3. Questions

Review: Setting

Geanakoplos (2010) model with risk aversion, no short-sale constraints:

- Single risky asset with single payoff at T, (Lucas tree that bears no fruit until T, then dies), which depends on number of up moves $m \in \{0, \ldots, T\}$ of i.i.d. binomial tree
  - Aside: Can be generalized by adding additional "assets" with payoffs at $1, \ldots, T-1, T+1, \ldots$
- Agent $h \in (0,1)$ believes probability of up move is h and "agrees to disagree" with other agents
  - Equivalent to learning with point-mass prior h (though what happens if we take $T \to \infty$?)
- Distribution of mass of agents is $h \sim \text{Beta}(\alpha, \beta)$
- Normalize risk-free rate to zero (e.g., by setting exogenous intermediate consumption appropriately)
  - Risk-free asset in zero net supply, and risk-free borrowing must be risk free (collateralized)
- Log utility over terminal wealth $\Longleftrightarrow$ myopic portfolio choice, so each agent solves

$$\max_{\text{shares}_{h,t}} h \log\Big(\underbrace{\text{wealth}_{h,t} - \text{shares}_{h,t} p_t + \text{shares}_{h,t} p_{\text{up},t+1}}_{\text{wealth}_{h,\text{up},t+1}}\Big) + (1-h)\log\Big(\underbrace{\text{wealth}_{h,t} - \text{shares}_{h,t} p_t + \text{shares}_{h,t} p_{\text{down},t+1}}_{\text{wealth}_{h,\text{down},t+1}}\Big)$$

Review: Basic Results

Optimality + market clearing (with some neat algebra using the payoff approach) give:

1. Wealth distribution: Fraction of aggregate wealth p_t held by type-h agents, $\frac{\text{wealth}_{h,t} f(h)}{p_t}$, follows Beta$(\alpha + m, \beta + t - m)$, where m is # of up moves from 0 to t
   - Why? Because the beta distribution is the conjugate prior of the binomial distribution, and have assumed beta "prior" distribution of agents and binomial evolution of tree
   - So beta distribution is the "right" choice for initial wealth distribution
   - Delivers very clear, closed-form generalization of logic of Geanakoplos (2010): wealth accrues to investors who are correct in hindsight

2. Pricing: At any date t, after m up moves, the risky asset's price $p_{m,t}$ is

$$p_{m,t} = \frac{1}{\sum_{m'=0}^{T-t} \text{Prob}_{\text{RepAgent},t}[(\text{up moves from } t \text{ to } T) = m'] \times p_{m+m',T}^{-1}}$$

   - This is "just" the harmonic-mean payoff perceived by the (wealth-weighted) rep. agent
   - Why harmonic mean? Because of log utility
   - What beliefs does this representative agent hold? More interpretation in a few slides, but note that bad news is amplified by pessimists becoming wealthier (& vice versa), *and* this is priced

Additional Results and Implications

(i) For very general payoffs as function of # of up moves, $p_{m,T}$, the risky asset's expected return is increasing in belief heterogeneity $\Longrightarrow$ Equity premium ✓

(ii) In good times (as the wealth-weighted avg. belief increases), *all* individual investors believe the market's Sharpe ratio is lower, but it can be shown that $\frac{d\text{SR}_{\text{RepAgent},t}}{d\text{RepAgentBelief}_t} > 0$ (should include this!)

- So while all individual investors underreact to new information (by design), the market overreacts to good news in the sense that it perceives a higher Sharpe ratio in good times $\Longrightarrow$ survey evidence [Bordalo, Gennaioli, Ma, Shleifer (2018)] ✓
  & excess volatility of prices and Sharpe ratios [Shiller (1981), . . ., Lazarus (2018)] ✓

(iii) Term structure of expected returns (as perceived by all agents) is downward sloping, with greater downward slope in bad times $\Longrightarrow$ term structure and cyclicality of risk premia ✓

- Same for term structures of implied and physical volatility

(iv) And all of this with a constant risk-free rate

Outline

Interpretation of Results: Decompositions

- Multiple new modeling choices here relative to previous benchmarks; how much does each contribute to results? Will focus just on prices
- Across all models under consideration:
  - Normalize $\mathbb{E}_0[p_{m,T}] = e$ (where expectation is w.r.t. representative agent's beliefs)
  - Assume agents are symmetrically distributed around up-move belief $h = 1/2$
    (all equal to $h = 1/2$ in homogeneous-agent case, and $\alpha = \beta = \theta N$ in heterogeneous case)
  - Work in continuous-time limit
- My benchmark model: Homogeneous risk-neutral economy, $p_0 = \mathbb{E}_0[p_{m,T}] = e$
- **Decomposition 1:**

$$\log\left(p_{0,\text{heterogeneous}} / p_{0,\text{benchmark}}\right) = \log\left(p_{0,\text{heterogeneous}} / e\right) = \log\left(p_{0,\text{heterogeneous}}\right)$$

$$= \underbrace{\log\left(p_{0,\text{heterogeneous}} / p_{0,\text{homogeneous,risk-averse}}\right)}_{\text{effect of heterogeneity}} + \underbrace{\log\left(p_{0,\text{homogeneous,risk-averse}}\right)}_{\text{effect of risk aversion}}$$

$$= \underbrace{-\frac{1}{2\theta}}_{\text{heterogeneity}} + \underbrace{-\frac{1}{2}}_{\text{risk aversion}} \overset{\text{main calibration}}{=} -0.28 - 0.5$$

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$$= \underbrace{-\frac{1}{2\theta}}_{\text{heterogeneity}} + \underbrace{-\frac{1}{2}}_{\text{risk aversion}} \overset{\text{crisis calibration}}{=} -2.5 - 0.5$$

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$$\log(p_{0,\text{heterogeneous}}/p_{0,\text{benchmark}}) = \log(p_{0,\text{heterogeneous}})$$

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$\implies$ rep. agent's belief in heterogeneous economy is *equal* to belief held by single agent in a learning economy ("wisdom of the crowd")

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$\Longrightarrow$ further, we learn that in isolation, uncertainty and disagreement work in exactly the same direction here

Outline

Remaining Questions

- Mechanism requires full strategic sophistication w.r.t. other agents' beliefs
  - All agents know other agents' beliefs (and know that other agents know their beliefs, . . .) and agree to disagree
- This generates strong short-term speculation
  - *"Agents take temporary positions, at prices they believe to be fundamentally incorrect, in anticipation of adjusting their positions in the future"*
  - Again in general works to push prices down, expected returns up
- But what if people don't realize that everyone else has different beliefs ("disagreement neglect")? [Eyster, Rabin, Vayanos (2019)]
- Would seem to weaken the main mechanism
- But this is (mostly) a quantitative issue, and maybe the main mechanism needs to be weakened!
  - Giglio, Maggiori, Stroebel, & Utkus (2019) show that retail investors' stock portfolios are much less sensitive to individual beliefs than implied by this model

Remaining Questions

- I've emphasized interpretation of risky asset as aggregate market
- But framework in principle applies broadly wherever differences of opinion are important
- One example: Coastal real estate given differences of opinion over climate change
  - Here, "risky asset" is coastal real estate, and can study its properties relative to otherwise equivalent unexposed property ("safe," normalized expected return)
- There exist estimates [e.g., McAlpine & Porter (2018)] of the $ loss realized by coastal property owners to date by coastal county, relative to equivalent unexposed units
  - Combine with estimates of current $ value of properties likely to be underwater [Union of Concerned Scientists (2018)] to find that just $\sim 1 - 5\%$ of expected losses as of 2100 have been impounded into current prices
  - Should pin down average optimism relative to heterogeneity in this market (η / θ), and term structure likely pins down the two separately
  - Normatively important question!

Final Notes

- Elegant and very useful paper
- Tractable framework for analyzing belief heterogeneity in a dynamic economy, with explicit solutions to many of the literature's big questions. . .
- . . . and opens up many questions of its own