

# The Size-Power Tradeoff in HAR Inference: Supplement

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This supplement provides additional figures and Monte Carlo results.

Figure S.1 plots the implied mean kernel for the Fourier, cosine, and SS basis functions for  $B = 8$  and  $T = 200$ . The Fourier transforms of these implied mean kernels, that is, the frequency-domain implied mean kernel, are plotted in Figure S.2 at the frequencies  $2\pi j/T, j = 0, \dots, 35$ . The EWP (Fourier) estimator is the only one of these four that has an exact kernel representation, and its frequency-domain kernel is the familiar flat (Daniell) kernel that gives equal weight to the first  $B/2$  periodogram ordinates. The remaining three implied mean kernels in the frequency domain also concentrate their mass at low frequencies.

Figure S.3 shows the power difference, as a function of the local alternative index  $\delta$ , between the EWP and QS test, for  $B = 8$  for EWP and  $b$  for QS chosen so that the two tests have the same size. This curve is computed using the expression in Theorem 3 and Remark 6.

Figure S.4 shows additional Monte Carlo results for different values of  $T$  for 6 tests: QS, EWP, cosine (type II cosine basis function), NW, and SS.

Figure S.5 shows the spectral density for the ARMA(2,1) process. The parameters are calibrated so that  $\omega^{(2)} = 4$  (the same as an AR(1) with  $\rho = 0.5$ ) and with a spectral density approximately symmetric around  $\pi/2$ , with a minimum at  $\pi/2$  (the coefficients are  $\rho_1 = 0.048, \rho_2 = 0.248, \theta = -0.064$ ).

Figure S.6 shows results for ARMA(2,1) disturbances,  $m = 1$ .

Figures S.7 and S.8 show additional results for  $m = 2$ .

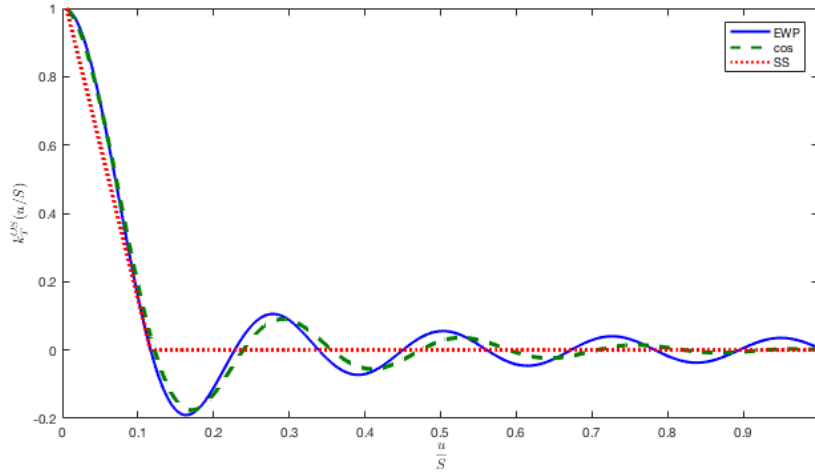


Figure S.1. Implied mean kernel of basis function estimators with  $B = 8$ , time domain: Fourier/EWP (blue, solid), cosine (green, dash), and split-sample (red, dot).

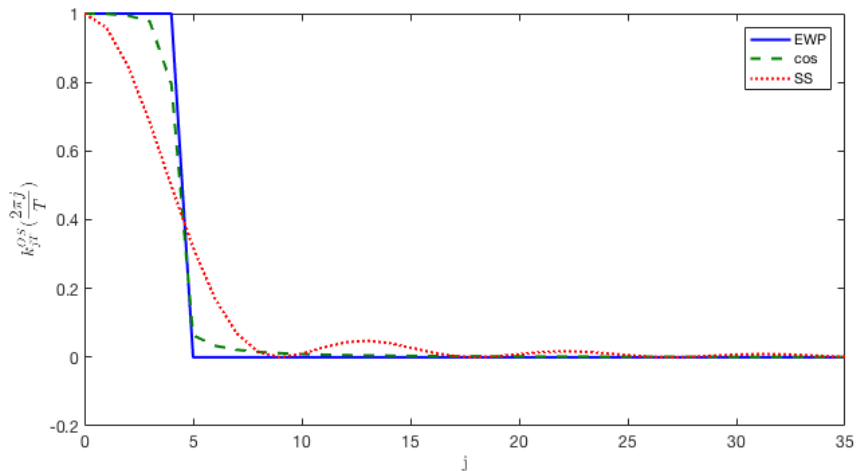


Figure S.2. Implied mean kernel of basis function estimators with  $B = 8$ , frequency domain: Fourier/EWP (blue, solid), cosine (green, dash), and split-sample (red, dot). The frequency domain kernel is normalized to 1 at  $\omega = 0$  and computed over the periodogram ordinates (so the horizontal axis value  $j$  corresponds to  $2\pi j/T$ , etc.).

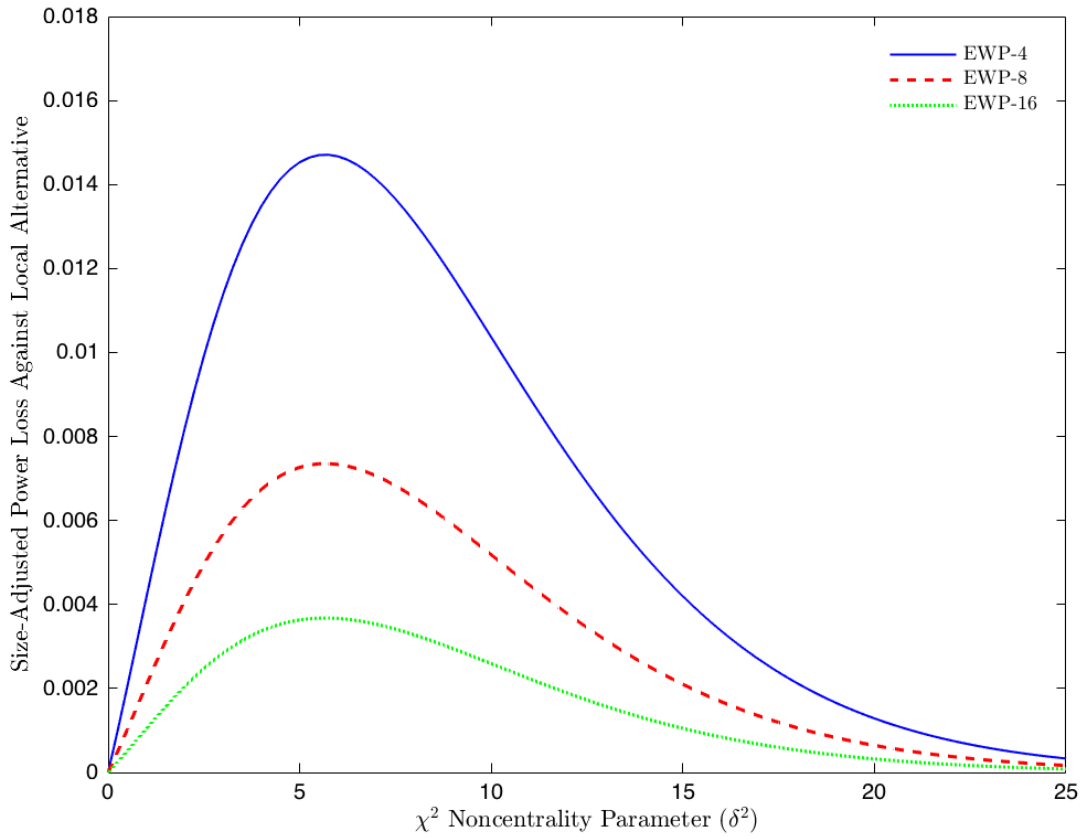


Figure S.3. Small- $b$  approximation to power loss for EWP test, compared to QS test, for different values of  $B$  in the EWP test and with  $b$  for the QS test chosen so that the EWP and QS tests have the same higher-order size when evaluated using fixed- $b$  critical values. The figure plots the final expression in (44) as a function of  $\delta$ . Gaussian location model,  $m=1$ , 5% significance level.

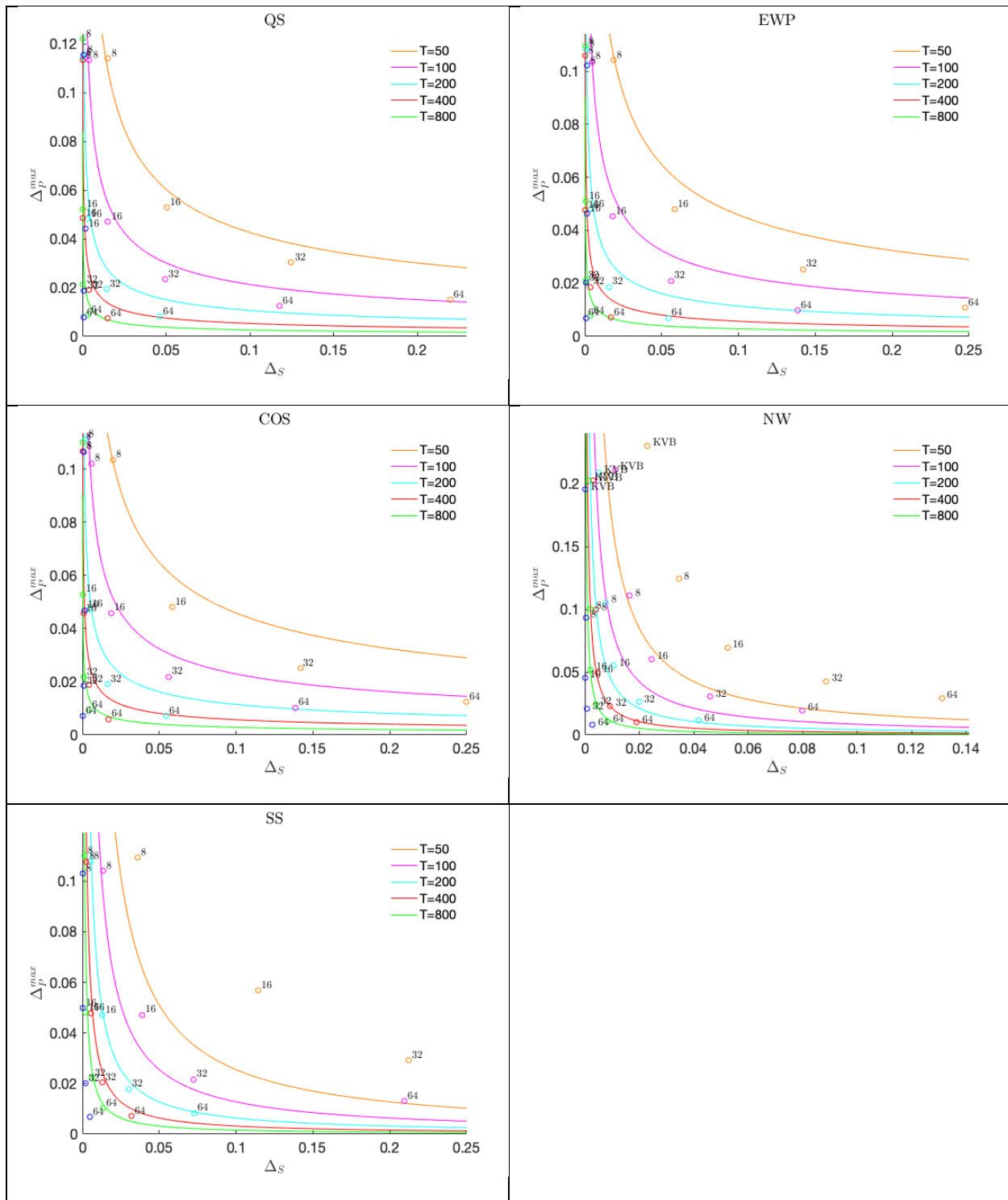


Figure S.4. Location model, AR(1),  $m = 1$ ,  $\rho = 0.5$ .  
 Theoretical size distortion/power loss trade-off curves for each estimator with Monte Carlo results for  $T$  ranging from 50 to 1600.

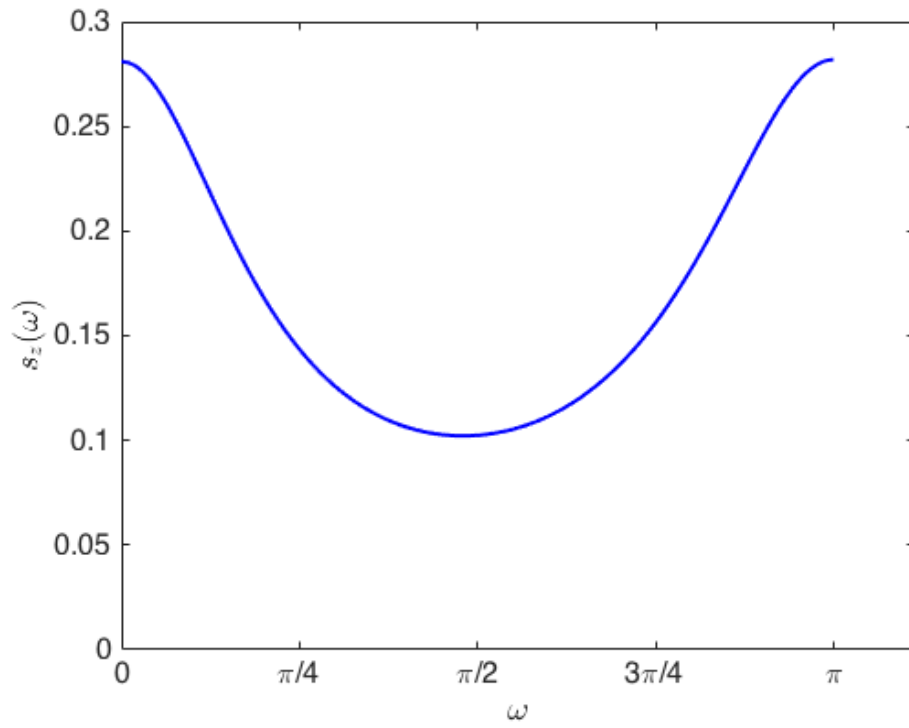


Figure S.5. Spectral density of calibrated ARMA(2,1),  $\omega^{(2)} = 4$ .

### ARMA(2,1) Model

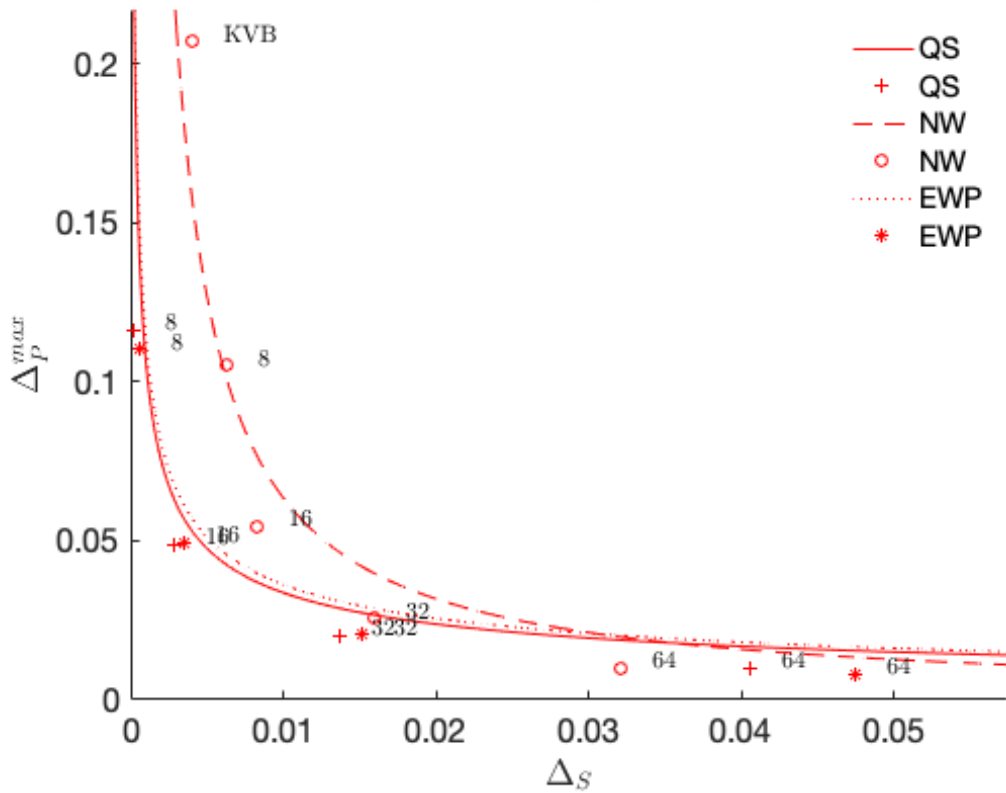


Figure S.6. Location model, ARMA(2,1),  $m = 1$ ,  $T = 200$ .

Theoretical size distortion/power loss trade-off curves for QS, Newey-West, and EWP estimators with Monte Carlo results. ARMA(2,1) parameters fixed such that  $\omega^{(2)} = 4$ , equivalent to AR(1) with  $\alpha = 0.5$  (parameter values as in Figure S.5).

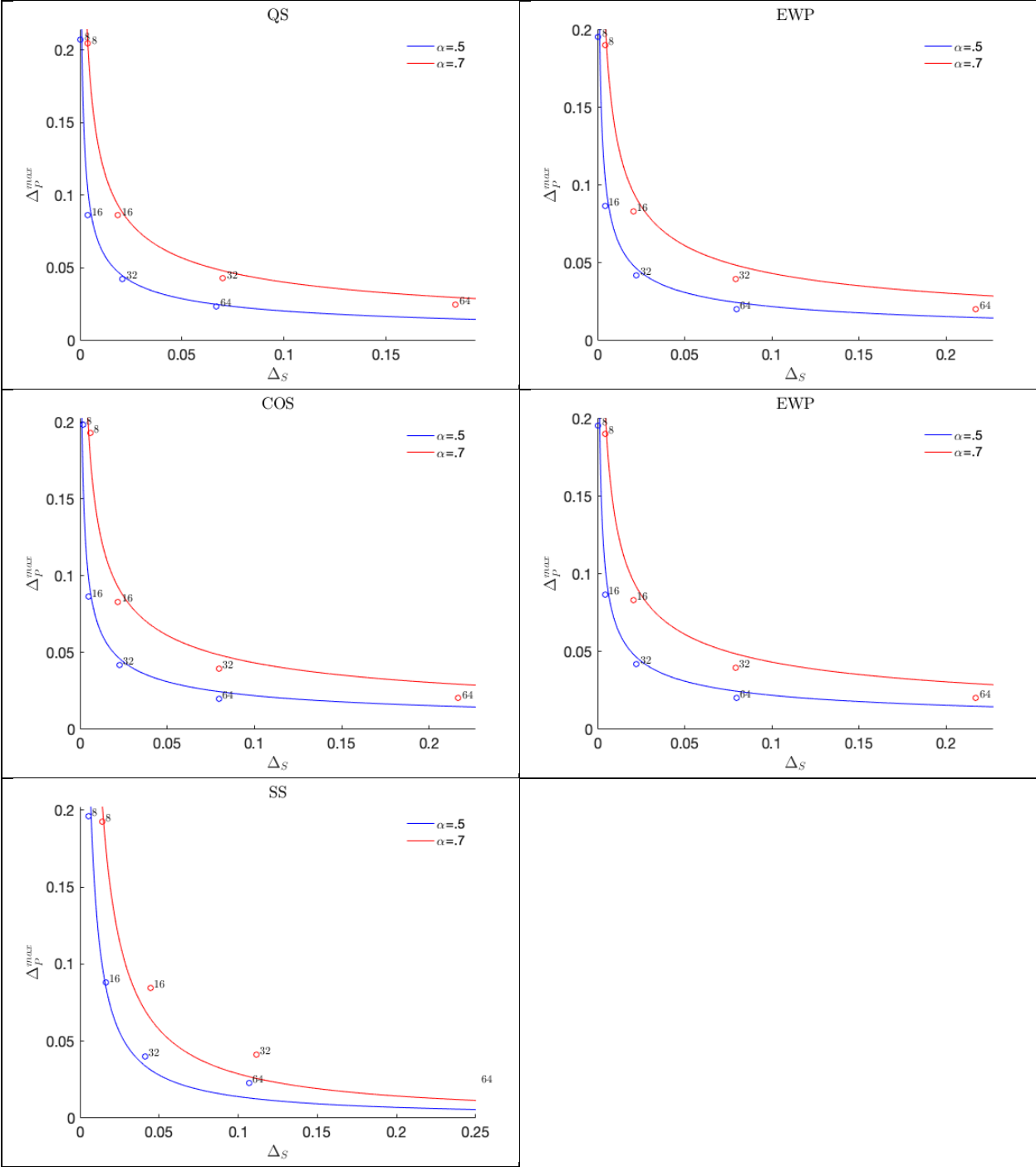


Figure S.7. Location model, AR(1),  $m = 2$ ,  $\rho = .5$  and  $.7$ ,  $T = 200$ . Theoretical size distortion/power loss trade-off curves for each estimator and Monte Carlo results (dots).

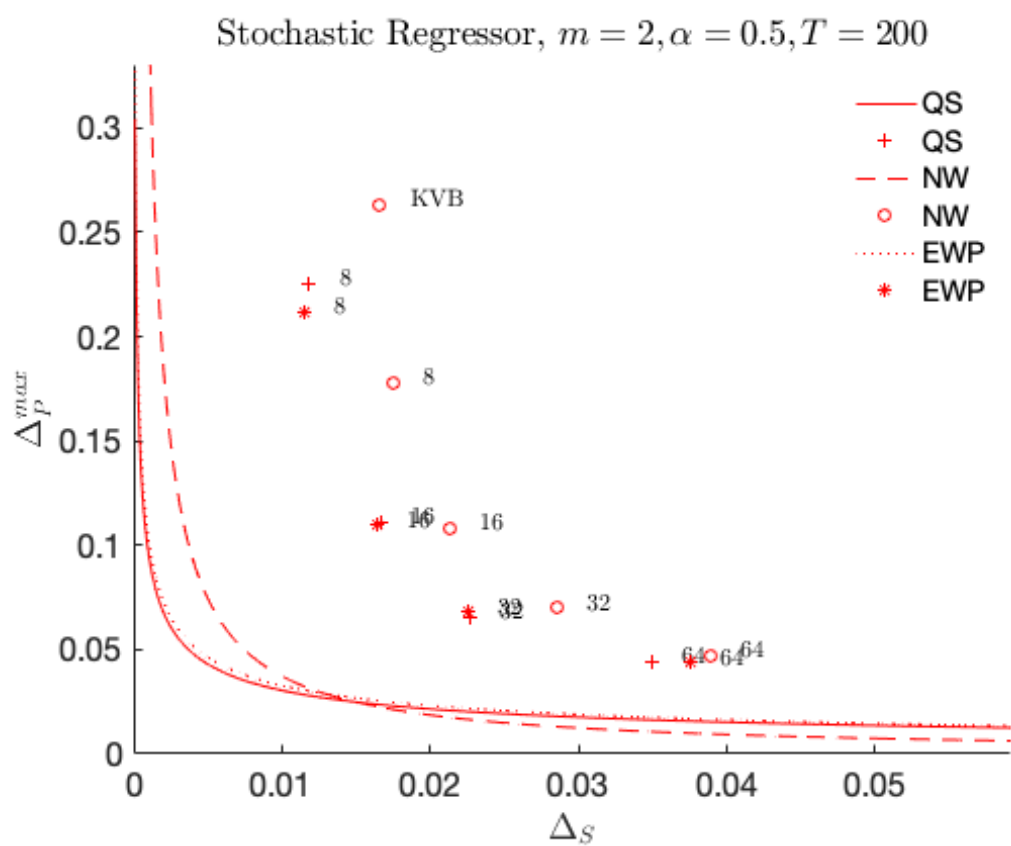


Figure S.8. Stochastic regressor, AR(1),  $m = 2, \rho = 0.5, T = 200$ . Theoretical size distortion/power loss trade-off curves for QS, Newey-West, and EWP estimators with Monte Carlo results. *Note:* Curves are for the Gaussian location model.