Model-Free International
Stochastic Discount Factors

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Discussion:
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Background

Exchange-rate puzzles:

1. Forward premium puzzle

2. With complete markets, exchange rates are “too smooth” unless we think risk-sharing is nearly perfect [Brandt, Cochrane, Santa-Clara (2006)]
   - For any foreign (non-US) asset with return $R_{f,i}$,
     \[
     \mathbb{E}[M_f R_{f,i}] = 1, \\
     \mathbb{E}[M_d X R_{f,i}] = 1,
     \]
     where $X \equiv S_{t+1}/S_t$ is exchange-rate change ($\uparrow$ means foreign appreciates relative to US), $M_d$ is US-investor SDF
   - With unique SDFs, $X = M_f / M_d$, so $\text{Var}(x) = \text{Var}(m_f - m_d)$
   - We know $\text{Var}(m_f), \text{Var}(m_d)$ are very high (equity premium + forward premium $\iff$ high Sharpe ratios available), so they must covary strongly given exchange-rate volatility of 15%/year

3. Cyclicality puzzle [Backus, Smith (1993)]
   - Exchange rates don’t comove empirically with proxies for relative macro conditions, even though (from above) $\text{Cov}(x, m_f - m_d)/\text{Var}(x) = 1$
Background

Standard approach to rationalize puzzles:

▶ Assume the existence of some “dark matter”

▶ For example, highly correlated long-run risks imply large Sharpe ratios, smooth exchange rates, and exchange rate comovement with hard-to-measure expectations of long-run consumption growth

This paper’s approach:

▶ Step back from strict parameterizations of preferences and fundamentals

▶ Instead, consider what we learn by semiparametrically characterizing certain SDFs under different assumptions about market segmentation
What I’ll do

Interesting and important set of questions

Discussion: Review step by step, with short comments/questions as I go

1. Theory
2. Empirical implementation
3. Results
Outline

1. Theory
2. Empirical Implementation
3. Results
Theoretical Setting

Work under null of:

1. Incomplete markets
   - Non-unique SDFs, so get an additional degree of freedom ("wedge") in matching exchange-rate returns [Backus, Foresi, Telmer (2001)]
     \[ x = m_f - m_d + \eta \]
   - Wedge isn’t unrestricted (e.g., orthogonal to asset returns)

2. Integrated markets:
   \[ \text{Span(domestic returns)} = \text{Span(foreign returns } \times \text{ exchange-rate change)} \]
   - But I thought we wanted to know what happens when markets are segmented?
   - Response: By characterizing set of SDF processes under integrated markets, can hope to draw (contrapositive) conclusions about necessity of segmented markets if those processes are “unreasonable”
Theoretical Setting

Toolkit:

1. Solve for minimum-dispersion SDFs
   - This problem is a bit convoluted — $\min_{M} \log \mathbb{E}[M^\alpha] / (\alpha(\alpha - 1))$. subject to pricing equation
   - But in practice, the authors consider just two such solutions: (i) minimum entropy ($\alpha = 0$), and (ii) minimum variance ($\alpha = 2$)
   - Proposition 1 gives “cookbook” for doing so given observed returns & $\alpha$
   - Important: Minimum-variance SDF is the unique SDF in return space $\iff$ gives projection of “true” SDF onto return space
     - Why? $\mathbb{E}[MR] = 1 \iff \mathbb{E}[(M + \varepsilon)R] = 1$ if $\mathbb{E}[\varepsilon R] = 0$, so $\varepsilon = 0$ gives lowest-variance SDF and further has a unique solution [Cochrane (2005)]

2. Consider some restrictions on exchange-rate wedge $\eta$ for $\alpha = 0, \alpha = 2$
   - $\eta = 0$ for $\alpha = 0$: Even with incomplete markets, minimum-entropy SDF is inverse of growth-optimal portfolio, which can be expressed in either domestic or foreign currency, so we’re stuck with $x = m_f - m_d$
   - $\eta = 0$ for $\alpha \neq 0$ iff $\text{Span}(\text{domestic returns}) = \text{Span}(\text{foreign returns})$; otherwise, there are unspanned exchange-rate risks

3. Decompose SDF into permanent/transitory components
Theoretical Setting

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3. Decompose SDF into permanent/transitory components
Theory: Interpretation and Comments

Interpretation:

1. There are lots of (infinitely many) SDFs in incomplete markets. How to interpret the series of minimum-dispersion SDFs that the authors solve for?
   ▶ Partial answer: These solutions by design give us conservative estimates of the moment being minimized
   ▶ But what about the other moments? Are we over- or underestimating the correlation between domestic & foreign SDFs? The cyclicality of the wedge? …

2. “Unspanned” exchange-rate risks: Exchange rate fluctuates based on innovations to $M(\text{min. entropy}) - M(\text{min. variance})$ in both countries, which is orthogonal to traded returns
   ▶ Direction and economic intuition a bit unclear
   ▶ $X \uparrow$ when foreign unspanned risk is “worse,” in order to compensate domestic investors for taking on that risk?
Outline

1. Theory

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Empirics

- Integrated but incomplete markets benchmark is that investors have access to aggregate equity, 10-year ($\approx \infty$-year) bonds, and short-term bonds in all (8) countries
  - US is domestic, average of all others is foreign
- This seems substantive and important: minimum-variance SDF depends on highest attainable Sharpe ratio, which of course depends on the set of assets you allow people to trade (and on the sample)
  - Sophisticated investors can access nonlinear foreign portfolios using derivatives
  - What’s the covariance of state-price densities of domestic vs. foreign stocks? Would seem to give valuable information about shared risks
  - Instead, we’re left to decide what a “reasonable” amount of risk-sharing vs. wedge volatility is when explaining exchange rate smoothness
- Segmented alternative: each country’s investors trade in their own 3 assets, plus short-term bond in other countries
Sensitivity

Table 2. Properties of SDFs (Integrated Markets)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha = 0$) and Panel B for minimum variance SDFs ($\alpha = 2$), $i = d, f, j = d, f, i \neq j$. The SDFs are derived when international trading is unrestricted, i.e. the financial markets are integrated. There is a US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

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<tr>
<td><strong>Panel A: $\alpha = 0$ (minimum entropy)</strong></td>
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<tr>
<td>$E[M_i]$</td>
<td>0.982</td>
<td>0.973</td>
<td>0.982</td>
<td>0.990</td>
<td>0.982</td>
<td>0.991</td>
<td>0.982</td>
<td>0.980</td>
<td>0.982</td>
<td>0.966</td>
<td>0.982</td>
<td>0.973</td>
<td>0.982</td>
<td>0.956</td>
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<tr>
<td>Std($M_i$)</td>
<td>0.841</td>
<td>0.872</td>
<td>0.979</td>
<td>0.926</td>
<td>0.740</td>
<td>0.694</td>
<td>0.690</td>
<td>0.681</td>
<td>0.919</td>
<td>0.951</td>
<td>0.726</td>
<td>0.720</td>
<td>0.639</td>
<td>0.557</td>
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<tr>
<td>Std($M_i^2$)</td>
<td>0.120</td>
<td>0.122</td>
<td>0.120</td>
<td>0.061</td>
<td>0.120</td>
<td>0.091</td>
<td>0.120</td>
<td>0.068</td>
<td>0.120</td>
<td>0.107</td>
<td>0.120</td>
<td>0.111</td>
<td>0.120</td>
<td>0.091</td>
</tr>
<tr>
<td>Std($M_i^3$)</td>
<td>0.917</td>
<td>0.948</td>
<td>1.048</td>
<td>0.951</td>
<td>0.814</td>
<td>0.707</td>
<td>0.774</td>
<td>0.725</td>
<td>1.029</td>
<td>1.065</td>
<td>0.823</td>
<td>0.827</td>
<td>0.681</td>
<td>0.625</td>
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<tr>
<td>$\sqrt{\text{Entropy}(M_i)}$</td>
<td>0.684</td>
<td>0.703</td>
<td>0.795</td>
<td>0.753</td>
<td>0.687</td>
<td>0.636</td>
<td>0.604</td>
<td>0.585</td>
<td>0.732</td>
<td>0.702</td>
<td>0.618</td>
<td>0.616</td>
<td>0.581</td>
<td>0.519</td>
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<tr>
<td>corr($M_i^2,M_i'$)</td>
<td>-0.454</td>
<td>-0.498</td>
<td>-0.407</td>
<td>-0.233</td>
<td>-0.519</td>
<td>-0.155</td>
<td>-0.549</td>
<td>-0.502</td>
<td>-0.411</td>
<td>-0.636</td>
<td>-0.506</td>
<td>-0.607</td>
<td>-0.317</td>
<td>-0.634</td>
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<td>corr($M_i,M_j$)</td>
<td>0.992</td>
<td>0.989</td>
<td>0.989</td>
<td>0.985</td>
<td>0.992</td>
<td>0.994</td>
<td>0.994</td>
<td>0.981</td>
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<td><strong>Panel B: $\alpha = 2$ (minimum variance)</strong></td>
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<td>0.982</td>
<td>0.973</td>
<td>0.982</td>
<td>0.956</td>
</tr>
<tr>
<td>Std($M_i$)</td>
<td>0.739</td>
<td>0.754</td>
<td>0.873</td>
<td>0.834</td>
<td>0.699</td>
<td>0.658</td>
<td>0.639</td>
<td>0.622</td>
<td>0.776</td>
<td>0.791</td>
<td>0.659</td>
<td>0.655</td>
<td>0.600</td>
<td>0.535</td>
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<td>Std($M_i^2$)</td>
<td>0.120</td>
<td>0.122</td>
<td>0.120</td>
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<td>0.120</td>
<td>0.091</td>
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<tr>
<td>Std($M_i^3$)</td>
<td>0.803</td>
<td>0.824</td>
<td>0.930</td>
<td>0.853</td>
<td>0.763</td>
<td>0.670</td>
<td>0.711</td>
<td>0.659</td>
<td>0.839</td>
<td>0.874</td>
<td>0.733</td>
<td>0.735</td>
<td>0.632</td>
<td>0.595</td>
</tr>
<tr>
<td>corr($M_i^2,M_i'$)</td>
<td>-0.517</td>
<td>-0.587</td>
<td>-0.455</td>
<td>-0.268</td>
<td>-0.552</td>
<td>-0.169</td>
<td>-0.597</td>
<td>-0.564</td>
<td>-0.500</td>
<td>-0.775</td>
<td>-0.566</td>
<td>-0.683</td>
<td>-0.340</td>
<td>-0.665</td>
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- Disaggregated integrated-market results across countries
- Focus on second row in panel B: US columns show that estimated minimum SDF volatility ranges from 0.6 to 0.87 (increase of 45%) depending on the foreign country considered
Outline

1. Theory

2. Empirical Implementation

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Results

Results on exchange-rate puzzles:

1. Forward premium puzzle
   - Estimated SDFs price carry returns by design, so this gets taken care of “for free”

2. Exchange rate smoothness
   - With integrated markets, even with XR wedge (minimum-variance case), need nearly perfect correlation between domestic & foreign SDFs (perfect risk-sharing) to explain the data. (Holds in general?)
   - Segmented markets: Lower SDF volatility (almost mechanically); less SDF comovement; higher wedge
   - Is this a win for the segmented-markets model?

3. Cyclicality puzzle
   - Still have $\text{Cov}(x, m_f - m_d)/\text{Var}(x) \approx 1$
   - So the response to the puzzle here is either: (i) consumption is the wrong proxy for the SDF; (ii) it could be the right proxy at short horizons (captured as temporary component as SDF, which exhibits acyclicality w.r.t. $x$), but the permanent component is what matters
Final Notes

- Really interesting paper, getting at important questions
- Lots of other stuff (including on possible importance of intermediaries) I didn’t even have time to touch on!
- Would love more on interpretation of minimum-dispersion SDFs — should we be taking them literally?
- Some room for additional empirical tests with more assets