Current Topics in Finance

Week 2: The Term Structure of Risk and Return

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Background

The "classics" in asset-pricing puzzles:

1. Equity premium puzzle
   - Risk aversion looks too high!
   - Mehra and Prescott (1985), Hansen and Jagannathan (1991)
2. Risk-free rate puzzle
   - High risk aversion gives implausibly high and volatile risk-free rate
   - Weil (1989)
3. Equity volatility puzzle
   - Covered last week
   - Shiller (1981), many others

See Campbell (2003) for an excellent survey.

Lots of proposed solutions, and still a lively literature (that I've worked within!).

But the puzzles themselves are well known, and possibly boring.

Background

The good news:

- In recent years, lots of debate about new puzzles and findings: heterogeneity, household behavior, inequality, cross-sectional returns, arbitrage, . . .
- Today: puzzles related to the term structure of returns
- Intuitively, exposure to distant-horizon uncertainty seems "worse" than exposure to near-term uncertainty $\Longrightarrow$ long-horizon claims should command a premium (and have higher Sharpe ratios) relative to short-horizon claims
- Empirically, using claims to equity dividends at different horizons, seem to find the opposite!

The bad news:

- Runs counter to all the models the literature relied on to resolve the classic puzzles!
- But this is of course actually good news for you: Lots of work to be done figuring out why this is the case, and how robust it is

Outline

1. Background & Intro

2. Binsbergen, Brandt, and Koijen (*AER*, 2012)

3. Lazarus (2018)

4. Gormsen and Lazarus (2019)

BBK (2012): Background and Setup

- Paper that kicked off this literature
- Very simple:

$$\begin{aligned}
\text{Index price}_t &= \sum_{i=1}^{\infty} \mathbb{E}_t \left[\frac{M_{t+i}}{M_t} D_{t+i} \right] \\
&= \underbrace{\sum_{i=1}^{T} \mathbb{E}_t \left[\frac{M_{t+i}}{M_t} D_{t+i} \right]}_{\text{price of short-term asset} \equiv \mathcal{P}_{t,T}} + \underbrace{\sum_{i=T+1}^{\infty} \mathbb{E}_t \left[\frac{M_{t+i}}{M_t} D_{t+i} \right]}_{\text{price of long-term asset}}
\end{aligned}$$

- How to measure $\mathcal{P}_{t,T}$ (and by implication, long-term asset)? Use S&P 500 index options, since put-call parity for option of maturity T tells us:

$$c_{t,T,K} + Ke^{-r^f_{t,T}(T-t)} = p_{t,T,K} + \text{Index price}_t - \mathcal{P}_{t,T}$$

for calls & puts with strike K. Rearrange to solve for $\mathcal{P}_{t,T}$.

BBK (2012): Basic Results

Holding-period returns, volatility, Sharpe ratio for short-term asset ($R_{1,t}$) significantly higher than for index itself:

TABLE 1—SUMMARY STATISTICS

	$R_{1,t}$	$R_{1,t} - R_{f,t}$	$R_{2,t}$	$R_{2,t} - R_{f,t}$	$R_{SP500,t}$	$R_{SP500,t} - R_{f,t}$
Mean	0.0116	0.0088	0.0112	0.0084	0.0056	0.0027
	(0.0044)	(0.0044)	(0.0044)	(0.0045)	(0.0047)	(0.0047)
Standard deviation	0.0780	0.0781	0.0965	0.0966	0.0469	0.0468
	(0.0136)	(0.0136)	(0.0171)	(0.0171)	(0.0050)	(0.0050)
Sharpe ratio	0.1124	—	0.0872	—	0.0586	—
	(0.0520)	—	(0.0494)	—	(0.1058)	—
Observations	165	165	165	165	165	165

Notes: The table presents descriptive statistics of the monthly returns on the two trading strategies described in the main text. Block bootstrapped standard errors (blocks of 15 observations) of each of the moments are in parentheses. Sample period is February 1996 through October 2010.

- $R_{2,t}$ is short-term asset return calculated using "dividend steepener" (to avoid concerns about costly index replication)
- Both results suggest downward-sloping term structure of equity returns
- Basic results hold (esp. for Sharpe ratios) using direct measurement via dividend futures: Binsbergen, Hueskes, Koijen, Vrugt (*JFE*, 2013)
- Data for BBK: `https://www.aeaweb.org/aer/data/june2012/20101209_data.zip`

BBK (2012): Additional Results

1. The two-year short-term asset has an average price of $\sim$ 3% of the overall S&P 500 price $\Longrightarrow$ most of index value is in claims to long-horizon cash flows
2. Nonetheless, there's still significant excess volatility in the price of the short-term asset relative to dividends:

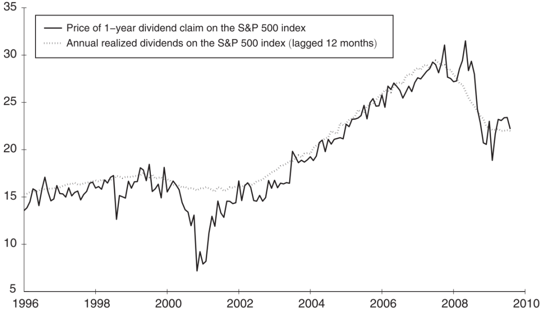


FIGURE 4. PRICES AND REALIZATIONS OF DIVIDEND CLAIMS: 1996:1–2009:10

- Relates to evidence from paper I presented to you last week
- As a corollary to point 2, short-term asset returns are highly predictable

Downward-Sloping Term Structure: Theory

What does existing theory have to say about these results?

- On the one hand, maybe this is intuitive from the standpoint of the well-known value premium
  - If growth (high book-to-market) stocks have long-horizon cash flows, then their low returns relative to value stocks line up with BBK's results [Lettau and Wachter (2007)]
- On the other hand. . .all workhorse models (long-run risks, habit, rare disasters) fail to reproduce BKK's results
  1. LRR: Bad consumption-growth shocks increase marginal utility and decrease future expected dividends, and this effect hits long-horizon claims most, so they have higher risk premia
  2. Habit: similar intuition
  3. Rare disasters: All growth is i.i.d., so term structure is flat, but Sharpe ratios are decreasing since volatility increases with maturity

Downward-Sloping Term Structure: Theory

What does existing theory have to say about these results?

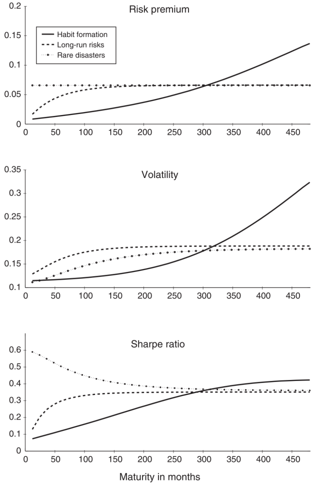


FIGURE 5. TERM STRUCTURE OF THE RISK PREMIUM, VOLATILITY, AND SHARPE RATIO FOR THE HABIT FORMATION, LONG-RUN RISKS, AND RARE DISASTERS MODEL

Notes: The graph shows the term structures of the risk premium, the volatility, and the Sharpe ratio for the Campbell and Cochrane (1999) habit formation model, the Bansal and Yaron (2004) long-run risks model, and the Gabaix (2009) rare disasters model. The graph plots the first 480 months of dividend strips, which corresponds to 40 years.

Downward-Sloping Term Structure: Theory

- Useful results: allow us to rule out certain features of existing theories
- But some concern about issues of measurement error, so how robust are the results?
- And from a positive standpoint, unclear where this leaves us
- Why does short-horizon equity give higher returns? Is it *riskier* per se, or do people just *dislike* the short-horizon risk more?
  - In asset-pricing jargon: is it *quantity* or *price* of risk that's driving the results?
- Will now cover my paper trying to address a small portion of these questions: "Horizon-Dependent Risk Pricing: Evidence from Short-Dated Options"

Outline

Background

How do people assess risky outcomes at different horizons?

- Recent literature [Binsbergen, Brandt, Koijen (2012), . . .]: Some evidence of downward-sloping term structure of equity returns, Sharpe ratios
  - **Problem 1:** Measurement error, short sample
    [Boguth et al. (2012), Bansal, Miller, Yaron (2017)]

    microstructure noise $\Longrightarrow$ upward bias of holding-period returns
    on short-term dividend strips
  - **Problem 2:** Interpretation? Downward-sloping price or quantity of risk?

    e.g., mean-reverting shocks (disaster + recovery) [Hasler and Marfè (2016)]
- Where I come in: Binary options over market index value at **short horizons**
  1. Already have data from previous paper I presented!
  2. Longer sample, and can address measurement error directly:
     buy-and-hold returns, plus instrument
  3. Provides additional info on source of declining risk premium:
     downward-sloping **price** of risk matters

Preview of Results

- Findings: Price of risk declines strongly with horizon
  - One-week horizon: RRA ≈ 15
  - 12-week horizon: RRA ≈ 3
- How can I calculate RRA from option returns?
  - Simple answer: I can't
  - More complete answer: I can look at strategies that fix riskiness of investment across horizons $\Longrightarrow$ tells us price of this risk

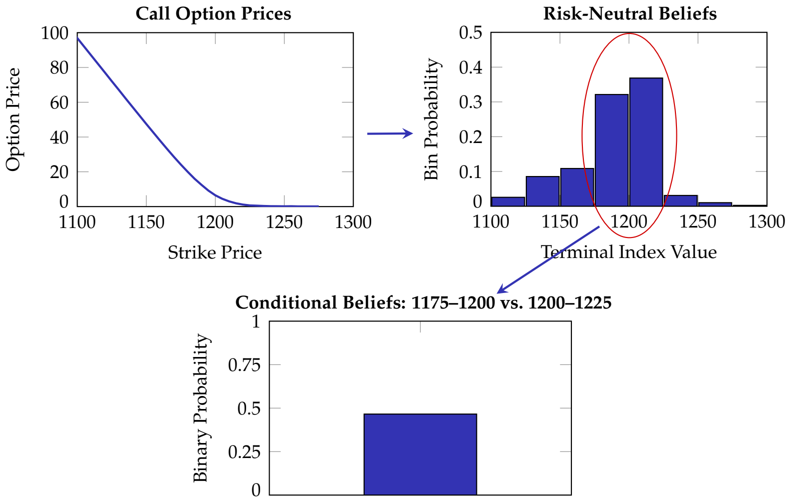
Preview of Results

- Findings: Price of risk declines strongly with horizon
  - One-week horizon: RRA ≈ 15
  - 12-week horizon: RRA ≈ 3
- I've fixed riskiness of investment only along one dimension. What if there are multiple sources of risk — is my interpretation still valid?
- Working under null that risk prices are in fact constant by horizon, derive necessary condition for rational framework to generate my results
  - Risk aversion over index return must *decrease* with marginal utility
  - Counterintuitive, difficult to meet this condition
- Lots of possible alternatives, but I'll discuss one in particular: dynamically inconsistent risk preferences (declining risk prices by horizon)
  - Can be viewed as reduced-form version of loss aversion + narrow framing, with narrow framing stronger as horizon becomes shorter

Estimation Setting: Empirical Illustration

S&P 500 Option Prices and Risk-Neutral Beliefs as of July 1, 2005

Expiration Date: July 16, 2005



Estimation Framework (I)

Notation and definitions:

- Index value V_t
- Index options begin trading on (normalized) date 0, expiration T
- Set (or subset) of possible index values at terminal date $\mathcal{V}_T \equiv \{v_1, \ldots, v_J\}$
  [assume discrete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geqslant 0}, \mathbb{P})$ for simplicity]
- $\pi_{t,j} \equiv \mathbb{P}_t(V_T = v_j \mid V_T \in \{v_j, v_{j+1}\})$
- Absence of arbitrage $\Longrightarrow$ risk-neutral measure $\mathbb{P}^*$, prob. $\pi^*_{t,j}$, where

$$\pi^*_{t,j} = \frac{\mathbb{E}_t[M_T \mid V_T = v_j]}{\mathbb{E}_t[M_T \mid V_T \in \{v_j, v_{j+1}\}]} \, \pi_{t,j},$$

  for strictly positive SDF process $\{M_t\}$

Estimation Framework (II)

Recall:

- $\pi_{t,j} \equiv \mathbb{P}_t(V_T = v_j \mid V_T \in \{v_j, v_{j+1}\})$
- $\pi^*_{t,j} \equiv \mathbb{P}^*_t(V_T = v_j \mid V_T \in \{v_j, v_{j+1}\}) = \frac{\mathbb{E}_t[M_T \mid V_T = v_j]}{\mathbb{E}_t[M_T \mid V_T \in \{v_j, v_{j+1}\}]} \pi_{t,j}$
- Rearranging,

$$\frac{\pi^*_{t,j}}{1-\pi^*_{t,j}} = \phi_{t,T,j} \frac{\pi_{t,j}}{1-\pi_{t,j}},$$

$$\text{where } \phi_{t,T,j} \equiv \frac{\mathbb{E}_t[M_T \mid V_T = v_j]}{\mathbb{E}_t[M_T \mid V_T = v_{j+1}]}$$

- Will be interested in how price of risk $\phi_{t,T,j}$ changes with horizon $T - t$
- Why use binary options? This price of risk can be viewed as (local) relative risk aversion for (fictitious) rep. agent who consumes terminal index value:

$$\gamma_{t,T,j} = \frac{\phi_{t,T,j} - 1}{(v_{j+1} - v_j)/v_j}$$

Estimation Framework (II)

Recall:

- $\pi_{t,j} \equiv \mathbb{P}_t(V_T = v_j \mid V_T \in \{v_j, v_{j+1}\})$
- $\pi^*_{t,j} \equiv \mathbb{P}^*_t(V_T = v_j \mid V_T \in \{v_j, v_{j+1}\}) = \frac{\mathbb{E}_t[M_T \mid V_T = v_j]}{\mathbb{E}_t[M_T \mid V_T \in \{v_j, v_{j+1}\}]} \pi_{t,j}$
- Rearranging,

$$\frac{\pi^*_{t,j}}{1 - \pi^*_{t,j}} = \phi_{t,T,j} \frac{\pi_{t,j}}{1 - \pi_{t,j}},$$

$$\text{where} \;\; \phi_{t,T,j} \equiv \frac{\mathbb{E}_t[M_T \mid V_T = v_j]}{\mathbb{E}_t[M_T \mid V_T = v_{j+1}]}$$

- Will be interested in how price of risk $\phi_{t,T,j}$ changes with horizon $T - t$
- For notational convenience, assume:
  1. *Scale independence*: if $v_{j+1}/v_j = v_{k+1}/v_k$, then $\phi_{t,T,j} = \phi_{t,T,k} = \phi_{t,T}$
  2. *Horizon-only dependence*: $\phi_{t,T} = \phi_{T-t}$

  Claim: Assumptions are innocuous; could just as well define $\phi_{T-t} \equiv \mathbb{E}[\phi_{t,T,j}]$, estimate by pooling over all dates t and state pairs j

Estimation Framework (III)

Moment condition:

- From above,

$$\pi_{t,j} = \frac{\pi^*_{t,j}}{\pi^*_{t,j} + \phi_{T-t}(1 - \pi^*_{t,j})},$$

so since $\pi_{t,j} = \mathbb{E}_t[\mathbb{1}\{V_T = v_j\} \mid V_T \in \{v_j, v_{j+1}\}]$ by definition,

$$\mathbb{E}_t\left[\mathbb{1}\{V_T = v_j\} - \frac{\pi^*_{t,j}}{\pi^*_{t,j} + \phi_{T-t}(1 - \pi^*_{t,j})} \,\middle|\, V_T \in \{v_j, v_{j+1}\} \right] = 0$$

  - Intuition: Estimating price of risk needed to reconcile ex-ante market forecast of terminal outcome with average observed outcome
  - No assumption on rationality of underlying subjective beliefs $\pi_{t,j}$

- Measurement error: $\widehat{\pi}^*_{t,j} = \pi^*_{t,j} + \epsilon_{t,j}$, where $\epsilon_{t,j}$ follows MA(q). Can then be shown that up to higher order,

$$\mathbb{E}_t\left[\mathbb{1}\{V_T = v_j\} - \frac{\widehat{\pi}^*_{t,j}}{\widehat{\pi}^*_{t,j} + \phi_{T-t}(1 - \widehat{\pi}^*_{t,j})} \,\middle|\, V_T \in \{v_j, v_{j+1}\} \right] = -\epsilon_{t,j}$$

GMM Estimation

$$\mathbb{E}_t\left[\mathbb{1}\{V_T = v_j\} - \frac{\widehat{\pi}^*_{t,j}}{\widehat{\pi}^*_{t,j} + \phi_{T-t}(1-\widehat{\pi}^*_{t,j})} \;\middle|\; V_T \in \{v_j, v_{j+1}\}\right] = -\epsilon_{t,j}$$

- Instrument vector: Lagged observations $Z_{t,j} = (\widehat{\pi}^*_{t-q-1,j}, \ldots, \widehat{\pi}^*_{t-\bar{q},j})'$
- Yields unconditional orthogonality condition:

$$\mathbb{E}\left[\left(\mathbb{1}\{V_T = v_j\} - \frac{\widehat{\pi}^*_{t,j}}{\widehat{\pi}^*_{t,j} + \phi_{T-t}(1-\widehat{\pi}^*_{t,j})}\,\mathbb{1}\Big\{V_T \in \{v_j, v_{j+1}\}\Big\}\right) Z_{t,j}\right] = 0$$

- Can now be estimated directly using GMM

Summary of Estimation

1. Options $\Longrightarrow$ market-implied (*risk-neutral*) distribution over index value at T
2. Transform into conditional probabilities: e.g., prob. that return will be between 6% and 8% conditional on being either in $[6\%, 8\%]$ or $[8\%, 10\%]$
3. π_t^* is distorted relative to π_t given risk aversion, but there is one-to-one relationship between these two values that depends only on effective risk aversion ϕ_{T-t}
4. π_t must be an unbiased forecast of the terminal index-return outcome, which is $\mathbb{1}\{\text{return is in } [6\%, 8\%] \mid \text{return is in } [6\%, 10\%]\}$. Since we observe π_t^*, can thus infer required ϕ_{T-t} such that underlying π_t is unbiased forecast.
5. Varying $T - t$ allows for estimation at different horizons, holding fixed the binary return outcomes (adjacent 2-ppt return bins)

Raw Data and Risk-Neutral Beliefs

Raw data:

- Want risk-neutral beliefs about return on market portfolio

$\Longrightarrow$ S&P 500 index option prices from OptionMetrics, 1996–2015 Details & cleaning

Measuring risk-neutral beliefs from options:

- Use standard techniques [Breeden & Litzenberger (1978)] to measure risk-neutral distribution for return state from option prices Details & robustness
- **Return space**:

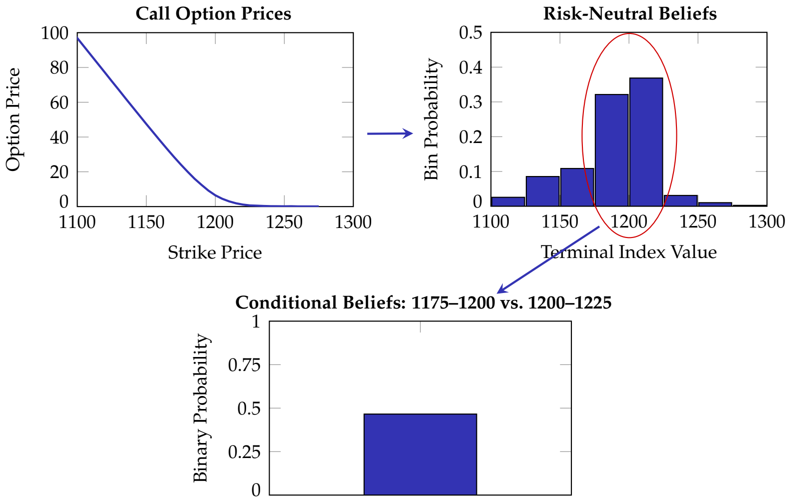
$$\text{Outcome}_T = \begin{cases} v_1 & \text{if } -0.10 \leqslant \overbrace{\log(V_T) - \log(V_0) - \log(R^f_{0,T})}^{\text{excess return from 0 to } T} \leqslant -0.08 \\ v_2 & \text{if } -0.08 \leqslant \log(V_T) - \log(V_0) - \log(R^f_{0,T}) \leqslant -0.06 \\ \vdots & \\ v_J & \text{if } \quad 0.08 \leqslant \log(V_T) - \log(V_0) - \log(R^f_{0,T}) \leqslant 0.10 \end{cases}$$

  - 2-ppt bins for excess returns
  - Exclude tail states (poorly measured, non-constant $\phi_{T-t,j}, \ldots$)
  - <200 observations beyond 12 weeks, so consider horizons 1-12

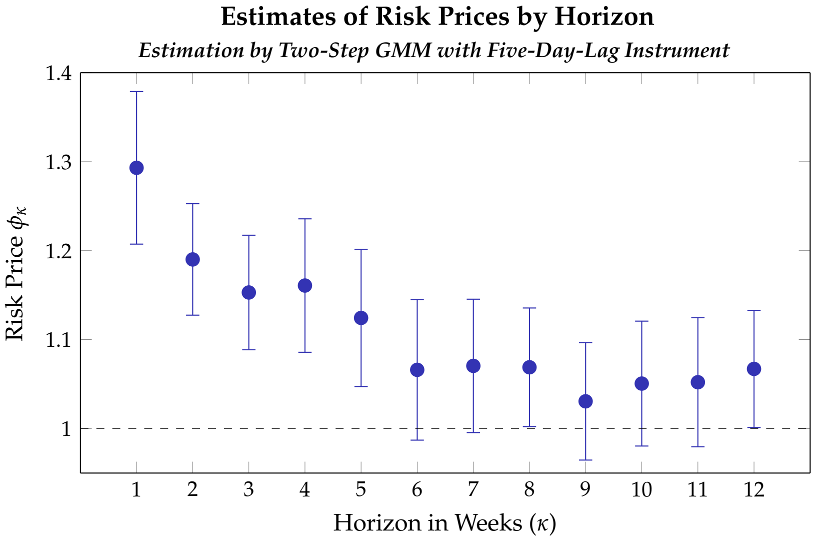
Reminder: Data Example

S&P 500 Option Prices and Risk-Neutral Beliefs as of July 1, 2005

Expiration Date: July 16, 2005

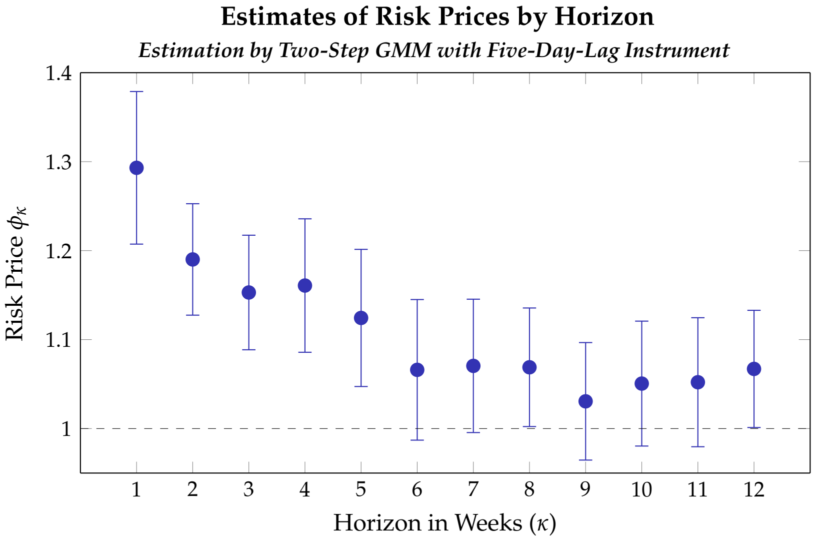


Main Estimation Results



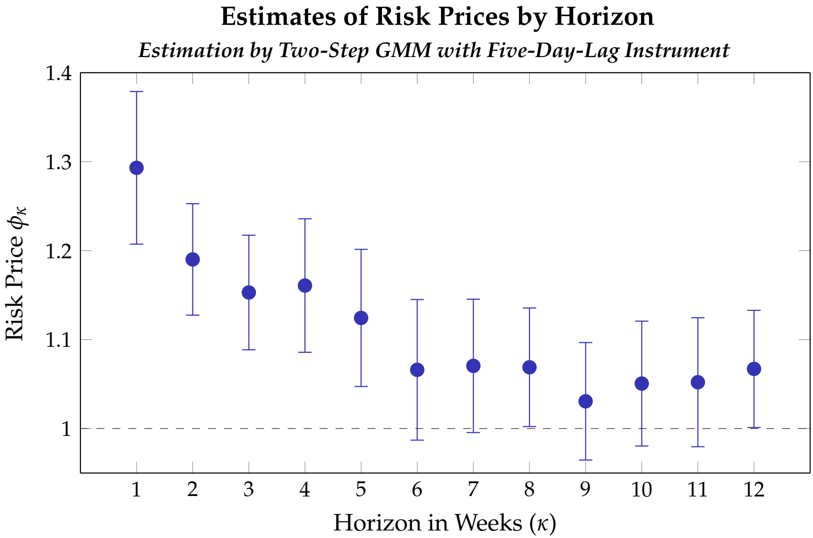
- RRA: 14.7 [10.4, 18.9] at one week, 3.4 [0.1, 6.6] at 12 weeks
- No apparent equity premium puzzle! [Bliss and Panigirtzoglou (2004)]

Main Estimation Results



- RRA: 14.7 [10.4, 18.9] at one week, 3.4 [0.1, 6.6] at 12 weeks
- J-statistic: $p = 0.30$ (HAR standard errors)

Main Estimation Results



- RRA: 14.7 [10.4, 18.9] at one week, 3.4 [0.1, 6.6] at 12 weeks
- Results robust to different instrument sets

Downward-Sloping Risk Pricing

To test more formally whether downward slope is robust, estimate regression:

$$\widehat{\phi}_\kappa = \alpha + \beta \kappa + \varepsilon_\kappa$$

- Null: $\beta = 0$
- Test using bootstrap: Estimate ϕ_κ on bootstrap samples, rerun regression in each sample, and calculate 95% CI as $[\widehat{\beta} - q^*(0.975), \widehat{\beta} - q^*(0.025)]$
  - $q^*(\cdot)$ is the quantile function of the bootstrap distribution of $\widehat{\beta}^* - \widehat{\beta}$
- Results:

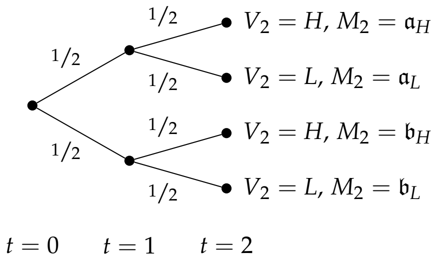
$$\widehat{\beta} = -0.018,$$

$$95\% \text{ CI } [-0.041, -0.007]$$

  - Downward-sloping term structure of risk prices

Interpretation: Standard Frameworks (I)

Consider simple 2×2 example:



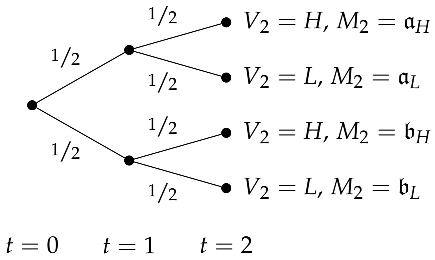
- Normalizations: $\mathfrak{a}_L > \mathfrak{a}_H$, $\mathfrak{b}_L > \mathfrak{b}_H$, $\mathfrak{b}_j > \mathfrak{a}_j$ for $j = L, H$
- $\phi_t \equiv \frac{\mathbb{E}_t[M_2 \mid V_T = L]}{\mathbb{E}_t[M_2 \mid V_T = H]}$, and want to know under what conditions $\phi_0 < \mathbb{E}_0[\phi_1]$
- Simple necessary & sufficient condition:

$$\frac{\mathfrak{a}_L}{\mathfrak{a}_H} > \frac{\mathfrak{b}_L}{\mathfrak{b}_H},$$

or $\phi_{1,\mathfrak{a}} > \phi_{1,\mathfrak{b}}$

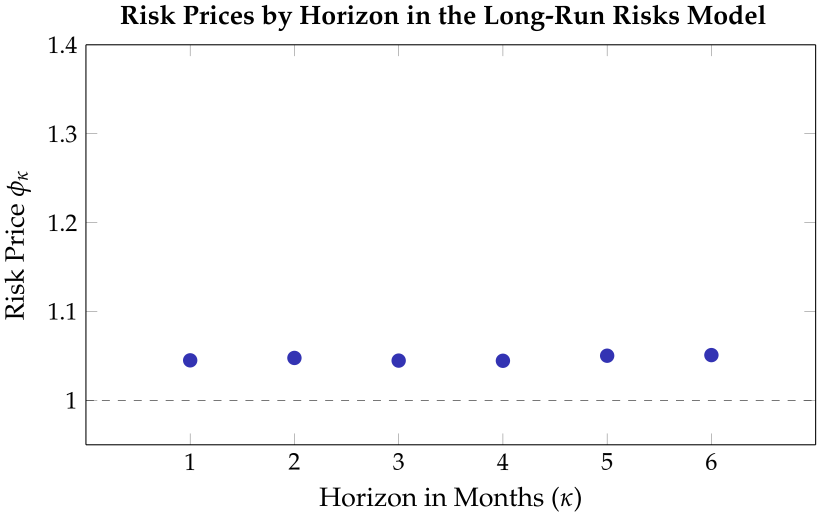
Interpretation: Standard Frameworks (I)

Consider simple 2×2 example:



- Normalizations: $\mathfrak{a}_L > \mathfrak{a}_H$, $\mathfrak{b}_L > \mathfrak{b}_H$, $\mathfrak{b}_j > \mathfrak{a}_j$ for $j = L, H$
- $\phi_{1,\mathfrak{a}} > \phi_{1,\mathfrak{b}} \implies$ risk aversion must be *higher* when agent receives *good* news in the sense that marginal utility is expected to be low
- Long-horizon gamble on good-state outcome H must be a good hedge against bad intermediate MU news
  - Not true in standard models (next slide: LRR)

Interpretation: Standard Frameworks (II)



Notes: Risk prices are calculated as averages over 2,000,000 years of simulated monthly data. The model and calibration are as given in Bansal & Yaron (2004, Case II). I solve the model numerically using the projection method of Pohl, Schmedders, & Wilms (2018).

Interpretation: Other Frameworks and Questions

Narrow bracketing and dynamically inconsistent risk preferences:

- Tversky and Kahneman (1981): Evidence of "narrow bracketing" of risks
- When do people narrowly frame? Results here seem to suggest it happens when horizon is short
  - Eisenbach & Schmalz (2016): Similar experimental evidence, suggesting dynamically inconsistent risk preferences as possible reduced-form modeling tool:

$$V_t = \left[(1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \mathbb{E}_t \left[\widetilde{V}_{t+1}^{1-\gamma_1} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma_1}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

$$\text{where } \widetilde{V}_{t+1} = \left[(1-\delta) C_{t+1}^{1-\frac{1}{\psi}} + \delta \mathbb{E}_t \left[\widetilde{V}_{t+2}^{1-\gamma_2} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma_2}} \right]^{\frac{1}{1-\frac{1}{\psi}}}$$

Final Notes

Summary:

- Present evidence of declining term structure of risk prices . . .
  - . . . w.r.t. gambles over small changes in market index value
  - . . . over short to medium horizons

Interpretations:

- Tough to rationalize in context of standard rep. agent models
- Have argued (vaguely) for time-inconsistent risk preferences, but could be:
  1. Belief biases: agents more pessimistic at shorter horizons
  2. Heterogeneity?

Other: Preferences over timing of resolution of uncertainty, interpretation relative to Augenblick and Lazarus (2018), other asset classes, . . .

Outline

1. Background & Intro
2. Binsbergen, Brandt, and Koijen (*AER*, 2012)
3. Lazarus (2018)
4. Gormsen and Lazarus (2019)

Intro

- Won't belabor the point on this paper too much, since it's early-stage and we're likely short on time (though see course site for draft)
- We do an exercise showing that many of the well-known cross-sectional return anomalies can be nearly fully explained by exposure to a cash-flow duration factor
    - High cash-flow growth (and thus high cash-flow duration) predict low returns in the cross-section, and vice versa
- Consistent with aggregate evidence from previous papers: short-term risk again seems to have a higher premium, and provides unifying framework for thinking about lots of seemingly separate cross-sectional puzzles
- We're not yet pointing to a structural/causal explanation
- But explaining the above evidence would bring together lots of disparate anomalies, both in aggregate and cross-sectional data

Main Empirical Finding

Relation between cash-flow growth and returns for 60 portfolios sorted on characteristics known to predict returns:

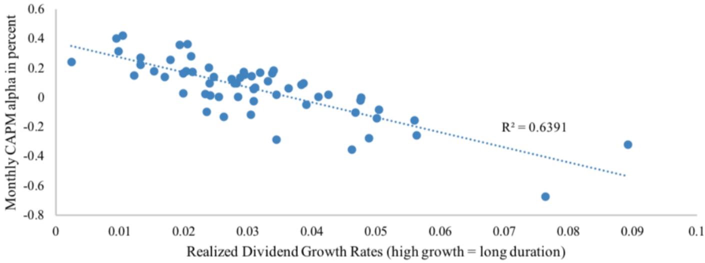


Figure 1: CAPM Alpha and Dividend Growth Rates for Characteristic-Sorted Portfolios

This figure shows the CAPM alpha and dividend growth rates for 60 portfolios sorted on profitability, investment, book-to-market, payout ratio, market beta, and idiosyncratic volatility. Alphas are estimated as the intercept in a monthly regression of excess return on the excess market returns. Alpha are in monthly percent and dividend growth rates are annualized dividend growth rates in the fifteen years following portfolio formation. The sample is the U.S. sample from 1963-2016.

Additional Empirical Results

- Results also hold using analyst expectations…
- … and an accounting measure of ex-ante duration…
- … and globally…
- … and when regressing cross-sectional characteristics directly on the above measures
- Then construct an equity "yield curve" by looking at book-to-market ratios of duration-sorted portfolios
  - Slope of this yield curve predicts 5-year aggregate market return with R^2 of 40%

Theory

- Don't yet have a structural/microfounded explanation
  - Currently working on explanation based on expectation errors
- Current theory: Reduced-form model for SDF as in Lettau and Wachter (2007), where only dividend risk is priced
  - As long as we assume that there's long-run insurance in dividend growth — i.e., negative contemporaneous shocks correlated with higher growth rates — we can recreate all our empirical findings

Table 1

Theory: Equity Risk Factors in a Model with a Downward Sloping Equity Term Structure

This table show the CAPM alpha and duration of equity risk factors in our asset pricing model. The CAPM alpha is the intercept in a regression of return to the risk factor on the market portfolio. Duration is measured the difference in value-weighted years to maturity of the cash flows of the firms in the long- and short leg of the risk factors. The table shows the median estimates of 1000 simulations of 700 quarters of data. Alphas are in monthly percent.

	SMB	HML	RMW	CMA	Low Risk	DUR
Long leg:	Small firms	High B/M	High profit	Low investment	Low beta	High duration
Short leg:	Big firms	Low B/M	Low profit	High investment	High beta	Low duration
CAPM alpha	-0.34	0.23	0.60	0.43	0.39	-1.35
Duration (years)	-8	-8	-8	-8	-8	12

Final Notes

- This is still a wide-open area, with lots of questions to be addressed
- Robustness of findings? Unifying different asset classes? Structural explanations in terms of preferences, technology, beliefs?
- Nice summary of state of existing literature, along with open questions and suggestions for future research: Binsbergen and Koijen (*JFE*, 2017)

Appendix for Lazarus (2018)

Raw Data: Details and Cleaning

Details of data:

- End-of-day prices for calls and puts, Jan. 1996–Aug. 2015, all traded strikes $\Longrightarrow$ 685 expiration dates T_i; 4,949 trading dates; 7,385,062 option prices
- Also obtain underlying index price, dividend yield, and risk-free rates from OptionMetrics, and hand-collect option settlement values from CBOE

Data cleaning:

- Drop any options with: bids of 0, Black-Scholes implied vol. more than 100%, greater than 6 months to maturity [Constantinides, Jackwerth, Savov (2013)], and any trading date–expiration date combos with fewer than 3 listed prices
- Calculate end-of-day price as average of listed bid and ask prices
- Cleaning for conditional risk-neutral probabilities: to avoid noisy measurement, only use date–state pairs meeting $\pi^*_{t,T_i,j} + \pi^*_{t,T_i,j+1} \geqslant 5\%$

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Spline Details

- Calculate $\frac{\partial}{\partial v} q_{t,T_i}(v)$ numerically following Malz (2014):
    1. Transform call and put price schedules for each date–expiration date set into Black-Scholes implied volatilities
    2. Fit clamped cubic splines to interpolate implied vols between strike prices for both calls and puts
    3. Average the calculated call and put implied vols at 1,900 strike prices
    4. Invert Black-Scholes implied volatility function to transform resulting implied vols back into call prices
    5. Numerically difference the resulting smoothed call-price schedule
- We only use Black-Scholes implied vols for smoothing and then transform vols back into prices, so doesn't require Black-Scholes model to be correct
- "Clamped" cubic spline: Sets slope of implied vol schedule to be zero at boundary strike-price values, and sets all implied vols below minimum observed strike price to value at minimum price (likewise for max.)
- This guarantees monotonically decreasing and convex call price schedule, which maintains no-arbitrage restrictions
- This is an *interpolating* spline: passes through all observed data (or *knot*) points

Alternative Smoothing Method

Bliss and Panigirtzoglou (2004) spline:

- Natural spline in implied vol–delta ($\Delta = \frac{\partial q}{\partial v}$) space, weighted by vega ($\nu = \frac{\partial q}{\partial \sigma}$)
- Smoothing spline: Penalizes squared second derivative of spline, with weight $\lambda = 0.01$ relative to deviations from observed data
- Force horizontal implied vol extrapolation by adding pseudo-data three strike intervals above/below observed strikes with implied vols equal to min./max. observed values before calculating spline
- Use out-of-the-money options (puts below current forward price, calls above)

In both cases, we set any negative probability values to 0 and renormalize the distribution accordingly. We calculate Bliss and Panigirtzoglou (2004) values to check robustness of main results.

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